A Model of R&D Tax Incentives

Eren Inci
Boston College Department of Economics, eren.inci.1@bc.edu

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Abstract

This paper examines R&D tax incentives in oligopolistic markets. We characterize the conditions both *analytically* and *graphically* under which the tax incentives serve as a tool to reach the socially desirable level of firm-financed R&D spending. Two basic results emerge from the model. First, it is not a *per se rule* that socially desirable level of R&D investment can be reached by imposing an R&D subsidy. When the sector spillover is sufficiently low, the government might want to *tax* R&D investments. This is, however, more likely to happen if the industry is highly concentrated. Second, the government's tax/subsidy policy works only if the sector is reasonably concentrated.

Keywords: R&D tax incentive, public policy, oligopoly, spillover

JEL Classification: H25; L13; O31

* *Tel.*: 1-617-552-3670; *fax*: 1-617-552-2308. *Address*: Boston College Department of Economics 140 Commonwealth Avenue Chestnut Hill MA 02467-3806 USA. *E-mail address*: inci@bc.edu. I am indebted to Richard Arnott for helping to focus the substance and direction of this paper. I would like to thank James Anderson, Arthur Lewbel and especially Richard Tresch, and participants of EEA-ESEM 2004 for their very helpful comments. A travel grant given by the European Economic Association for my EEA-ESEM presentation for this paper is gratefully acknowledged.

1 Introduction

Economic development in a growing number of countries has been dependent on R&D driven technologies. This is mostly because R&D and, more generally, *innovation* are considered the keys to productivity increases and high growth performance. In their seminal works, Solow (1957), and Denison (1974) showed that a significant portion of US productivity increase comes from technological change. Empirical analysis by the OECD (2001) affirms that R&D increases multi-factor productivity.

An increasing number of countries are offering special fiscal incentives to businesses to induce spending on R&D and increase their level of innovation. Gravelle (1999) estimated that the US R&D tax credit cost around $ 2.24 billion in lost revenue over fiscal years 1997 through 2002. A survey of the European Commission (1995) on state aid reports that its members spent over $ 1 billion per annum on R&D tax incentives during the early 1990's. In Canada, due to R&D tax incentives, the after-tax cost of R&D expenditure ranges between 35 and 50 cents per dollar spent depending on the type of firm and the province in which the R&D activity is conducted. In many other major countries' governments are trying to stimulate and encourage the creation of new technical knowledge. These are just a few examples that show the importance of public policy towards R&D investments[1].

Most papers analyzing the policy issues associated with R&D investments are empirical. Some earlier studies, mostly by Mansfield, claim that R&D tax incentives were more a burden than a stimulator. Mansfield (1986) found that R&D is not very sensitive to changes in its after-the-tax price. More recent studies, including Hall (1993), found that the price elasticity is at least unity. In a related vein, Dagenais et al. (1998), and Bernstein (1986, 1998) studied R&D in Canada. Asmussen and Berriot (1993) studied R&D in France and the Australian Bureau of Industry Economics (1999) studied R&D in Australia. All these studies tend to find price elasticities larger than one, meaning that R&D tax incentives are

[1]See Hall (1993, 1995) for a review of some of the history, regulations and methodology related to tax credits in the US and other OECD countries.

working[2]. We know that different empirical approaches produce different results. Regression estimates using time series data show considerable effects of R&D credits on R&D spending, whereas some indirect measures, such as the ones based on the price elasticity of demand for R&D, yield more modest results. Is the debate among empirical economists caused mainly by the differences in empirical techniques or is there any room for fundamental reasons?

Despite the debates over the effectiveness of R&D tax incentives in the empirical literature, somehow, public policy towards R&D investments has been under estimated by the applied microeconomists. Our paper, among other a few, is an attempt to fill the gap in this research area. From the theoretical point of view, it is usually believed that R&D investment has some characteristics of a public good. Therefore, it can be considered *partially* nonexcludable and nonrivalrous (See Arrow (1962), and Romer (1990)). It is *partially* nonexcludable because no one has the ability to exclude others from taking advantage of it, unless there is absolute intellectual property protection[3]. It is nonrivalrous since it does not wear out or suffer from congestion. Hence, we expect the output of R&D investment to have the properties of a public good. The only difference is that it is mostly produced privately. Given this evidence, we should consider some incentive mechanisms to induce R&D investment. "Who should do this?" is not at the center of the disagreement. There is no controversy over the intervention of government into the R&D businesses and government's role in encouraging appropriate R&D activity levels. However, with this paper, we would like to start a debate on how this should be done under different conditions.

This paper contributes to the literature in two ways. First, we set up a variation of the well-known D'Aspremont-Jacquemin (1988) model from which the optimal R&D policies are derived. We show that it is not a *per se rule* that the socially desirable level of firm-financed R&D investment can be reached by imposing a tax credit. In some cases, it is in society's best interest to tax R&D investments, and the tax does not necessarily result in a lower equilibrium level of R&D investment. Second, we derive results in such a way that we

[2] See Hall and Van Reenen (1999) for a survey of the empirical analysis of the R&D tax incentives.

[3] E.g., via patents with no knowledge spillovers within and across the industries.

can present the policy actions both analytically and graphically. We believe our graphical representation increases the intuition provided up to now.

The paper is organized as follows. Section 2 defines the model. Section 3 characterizes the R&D investment for a regulated oligopoly. Section 4 discusses the welfare issues in the decentralized equilibrium. The direct optimum is also given here. Section 5 concludes and summarizes the paper. Last section is an appendix that contains some detailed derivations and proofs.

2 The Model

We set up a three-stage game theoretic model in which we have a representative consumer, n firms, and a social utility maximizing government. We assume an oligopolistic market in which firms engage in output competition à la Cournot. Therefore, we immediately expect there will be under production due to the market power.

In the first stage of the game, the government commits to its optimal tax/subsidy policy towards output and R&D investment. In the second stage, firms decide how much to spend on R&D. In the final stage, they decide how much to sell in an oligopolistic market. We allow for the possibility of imperfect appropriability in the form of inter-firm spillovers.

2.1 Consumers

Following Dixit (1979), we assume there is a representative consumer whose preferences are given by a money metric utility function which is additively separable and linear in the numeraire good m.

$$U(Q, m) = u(Q) + m \tag{1}$$

where Q is the total demand of the representative consumer for output. With this functional form, we have no income effects and therefore the discussion boils down to a partial equilibrium analysis. Suppose the utility from the total output is quadratic and takes the following form

$$u(Q) = aQ - b\frac{Q^2}{2} \qquad a, b > 0 \tag{2}$$

From the maximization problem of the representative consumer we get the linear inverse demand function:

$$p(Q) = a - bQ \qquad \frac{a}{b} \geq Q \tag{3}$$

where p is the price of the good[4].

2.2 Firms

Consider an oligopolistic industry that consists of n symmetric firms indexed by i. Firms produce a single homogeneous good. When they do not engage in R&D activities the marginal cost of production is constant and equal to c. We call this the *initial marginal cost*. When they engage in R&D activities, the marginal cost of production decreases. In accord with D'Aspremont and Jacquemin (1988), we assume R&D investments have a cost reducing behavior. A quality-enhancing behavior can also be assumed in a product differentiation model. In market economies, the level of cost reducing R&D is determined by profits. Since profits may understate the social benefits at the margin, there is no reason to believe ex ante that the market outcome is optimal.

For various reasons[5], we assume that the rival firms' R&D expenditures also reduce the firm's marginal cost, but by less than the firm's own R&D investment. In the model, this effect is captured by a technical knowledge spillover α within the industry. On one hand, spillovers reduce cost of production of one firm. On the other hand, they decrease incentives for new R&D investments since it helps competitors to decrease their costs. We assume that the marginal cost function is the same for all firms, and takes the following convex

[4] In the general form of the demand function, we need $p(Q)$ to be twice continuously differentiable and to satisfy $\partial p(Q)/\partial Q < 0$ for all $Q \geq 0$ such that $p(Q) \geq 0$.

[5] An example can be imperfect protection of intellectual property rights. See Mansfield (1985, 1992, 1994), and Lee and Mansfield (1996) for some discussion on spillovers.

form for $firm\ i$:

$$\Phi_i = \Phi_i(I_i, \boldsymbol{I}_{-i}; c, \alpha) \qquad \forall i \in \{1, 2, ..., n\} \text{ and } \forall \alpha \in [0, 1] \tag{4}$$

where I_i represents the R&D expenditure level of the $firm\ i$ and $\boldsymbol{I}_{-i}$ for all other firms. Φ_i is assumed to be twice continuously differentiable in I_i and $\boldsymbol{I}_{-i}$. Since we have spillovers within the sector, any industrial knowledge rapidly becomes a public good. This prevents firms from enjoying all the benefits of their own R&D investments. One functional form which captures this effect for $firm\ i$ can be[6]

$$\Phi_i = c - [(1 - \alpha)I_i + \alpha \sum_{k=1}^{n} I_k] \tag{5}$$

With n firms, inverse demand function can be rewritten as follows:

$$p(q_i, \boldsymbol{q}_{-i}) = a - b \sum_{k=1}^{n} q_k \tag{6}$$

which is a linear and therefore concave demand function. The convex form for marginal cost and concave form for the demand function assures the concavity of the profit function and hence the existence and the uniqueness of the equilibrium. Here, q_i is the output of $firm\ i$, and $\boldsymbol{q}_{-i}$ is the output from all other firms. We assume that R&D represents a cost that depends on the efficiency of the R&D activities. This has effects on marginal cost and allocative efficiency[7]. Some amount of subsidy is given for every dollar spent on R&D and output is taxed. The profit function for $firm\ i$ is, then, given by

$$\Pi_i(q_i, \boldsymbol{q}_{-i}, I_i, \boldsymbol{I}_{-i}) = [p(q_i, \boldsymbol{q}_{-i}) - t]q_i - q_i\Phi_i - (1 - s)\Psi(I_i) \qquad \forall i \in \{1, 2, ..., n\} \tag{7}$$

[6] This is also the average cost of production. For future purposes, marginal cost can also be written as $\Phi_i = c - [I_i + \alpha \sum_{k \neq i}^{n} I_k]$.

[7] Allocative inefficiency occurs when pricing deviates from marginal cost. It is easy to show that $p = \frac{\varepsilon}{n\varepsilon + 1}(\Phi_i + t)$ where ε is defined to be price elasticity ($\varepsilon = -\frac{d \log p(Q)}{d \log Q}$). Therefore price is a markup over after-tax marginal cost. This is similar to the mark-up that can be derived from a Dixit-Stiglitz (1977) type product differentiation model. As indicated by Spence (1984), what is more in R&D than product differentiation is the appropriability problem or in general an externality problem.

where t is the tax on output, s is the subsidy on R&D, and $\Psi(I_i)$ is the cost of R&D activities which is a function of R&D investment. Initially, we define $t < p(q_i, \boldsymbol{q}_{-i})$ and $s \in [0, 1]$, but we shall relax these assumptions without loss of any generality whenever an optimum occurs with a negative tax (*a subsidy on output*) or with a negative subsidy (*a tax on R&D*).

R&D investment might be considered the most important source of product differentiation in determining whether firms can earn excess profits. Moreover, since R&D investment is costly, it is going to be a very good way to deter entry. If a new entrant wants to compete effectively with the incumbents, it has to bear high R&D costs which make it difficult to enter that market. Therefore, in the R&D driven sectors we expect to see higher concentration ratios. Our model is, by no means, a product differentiation model, but it captures the latter effect. Moreover, we look at the sectors in which a substantial amount of production cost is affected by R&D investments. Markets are usually oligopolistic in such sectors. That is why we prefer to use an oligopoly model in this paper.

2.3 The Government

We are treating the government as an active player with full commitment powers to set the available policy tools. The government chooses the optimal policy mix (t^*, s^*) so as to maximize its social welfare function, which is to be defined later. Optimal policy comprises an output tax t to deal with market power in the output market and an R&D subsidy s to deal with R&D market failure.

As with any investment decision, R&D investment is not undertaken by firms unless it is profitable. They sometimes over-invest in R&D due to the strategic reasons. These make governments intervention crucial for optimality. By changing the relative cost of R&D for every given level of output to any other investment through the optimal tax/subsidy policy towards R&D investments, the government can influence the generation of technical knowledge in the second best sense. It is an effective tool to internalize the spillover effects.

Moreover, *if possible*, the government can use another tax/subsidy policy towards market power. These two policies addressing two market failures are sufficient to bring the economy to its first best frontier. Thus, as we will be showing later, a decentralized equilibrium with tax/subsidy policies by the government exactly matches with the optimum of a welfare maximizing social planner.

2.4 The Game

We are considering a three stage game in which the firms and the government interact strategically. Strategic interaction of the firms tends to reduce the output, which makes subsidies justifiable. We have infinite continuous choice spaces in all stages of the game.

In the first stage, the government decides on the linear tax/subsidy rate on the output and the tax/subsidy for R&D investment. Both together form the optimal policy mix (t^*, s^*).

In the second stage, firms decide their R&D investment levels. When firms act strategically by choosing R&D before they set their output level, they have a stronger incentive to invest more in R&D[8]. By doing so they gain a stronger competitive advantage over their rivals. However, externalities associated with technological spillovers smoothens this incentive. Therefore, when spillovers are high enough, we expect firms to underinvest in R&D in an unregulated market equilibrium since spillovers are decreasing the marginal costs of all firms.

In the final stage, firms decide how much to sell in the oligopolistic market by engaging in Cournot competition. In the absence of an optimal policy mix (t^*, s^*), there will be under production relative to the social optimum due to the imperfect competition.

We assume that each firm and the government know how its actions will change the actions of all the others in the next stages of the game, which requires subgame perfection. Moreover, for the sake of completeness, it is worth mentioning that our solution concept is backward

[8] We assume output levels can be adjusted more quicklly than R&D investment levels. Therefore, the output game is played in a later stage than the R&D game.

induction. Assume n is fixed, for example, because of entry and exit barriers. Therefore, we can treat the number of firms as a parameter from now on.

3 Characterization of R&D Investment in a Regulated Oligopoly

The maximization problem of the $firm\ i$ yields the reaction functions for the Cournot outputs[9]:

$$q_i = \frac{a - b\sum_{k\neq i}^{n} q_k - \Phi_i - t}{2b} \qquad \forall i \in \{1, 2, ..., n\} \tag{8}$$

We can get the Cournot-Nash equilibrium of the stage-game by solving these reaction functions simultaneously:

$$q_i = \frac{a - t - (n+1)\Phi_i + \sum_{k=1}^{n}\Phi_k}{b(n+1)} \tag{9}$$

Details are given in the Appendix A.1. By substituting (9) into the profit function (7) we get the profit function in the compact format:

$$\hat{\Pi}_i(q_i(I_i, \boldsymbol{I}_{-i}), I_i) = bq_i^2 - (1-s)\Psi(I_i) \tag{10}$$

The first order conditions for the second stage of the game are then,

$$\frac{\partial\hat{\Pi}_i}{\partial I_i} = 2b\frac{\partial q_i}{\partial I_i}q_i - (1-s)\Psi'(I_i) \qquad \forall i \in \{1, 2, ..., n\} \tag{11}$$

Therefore, the marginal benefit of R&D for $firm\ i$ is $2b\frac{\partial q_i}{\partial I_i}q_i$.

Observation 1: *In an oligopolistic market with technology spillovers, a firm's marginal*

[9] All the conditions guaranteeing nonnegativity are assumed from now on. In particular $a > \Phi_i > 0$.

benefit of R&D investment can be decomposed as follows.

$$(M)(TTC)(\frac{1}{RI}) = \underbrace{\frac{2}{(n+1)}}_{\text{mark-up}} \underbrace{\left(\overbrace{\sum_{k \neq i}^{n} (\eta_{\Phi_k : I_i} \Phi_k)}^{\text{inter-firm effect}} - n \overbrace{(\eta_{\Phi_i : I_i} \Phi_i)}^{\text{own effect}} \right)}_{\text{total technology change}} \underbrace{\frac{q_i}{I_i}}_{\text{inverse of research intensity}} \tag{12}$$

Derivation of Observation 1 is given in the Appendix A.3. Here, M is a mark-up and $\eta_{\Phi_i : I_j}$ is the elasticity of the marginal cost of $firm\ i$ with respect to the R&D investment of $firm\ j$. I_i/q_i is the R&D investment per unit of output which is defined to be *the research intensity* (RI) in the literature. The change in the marginal cost of production can be interpreted as the change in the production technology of the firm (Total Technology Change, TTC). As shown in (12), this technology change comes from two sources. First, the firm's investment in R&D decreases its own cost of production. This *own effect* is captured by the second term in the big parenthesis. Second, there is an external effect for the firm coming from its rivals. It is impossible for a firm to contribute only to its own technological accumulation without contributing to the others' because of the spillovers involved. Remember that in Cournot setting, relative technology change is more important than absolute technology change since all the firms are interacting strategically. Any technology investment creates a change in the technology of the other firms which is captured by the first term in the big parenthesis. We cannot say anything about the sign of this *inter-firm effect* unless we know the nature of the sector, especially the spillovers associated with technological improvements. Therefore, the positive externality for the other firms might have a negative feedback effect on the firm itself. However, intuitively, the positive externality is more likely to have a positive effect on the firm itself when there is high technological transmission in the sector. We shall pin down this result more in Proposition 1.

To be able to go further assume that the cost of R&D investment is given by

$$\Psi(I_i) = \gamma \frac{I_i^2}{2} \tag{13}$$

which means that there are diminishing returns to R&D. The parameter γ captures the efficiency of R&D activities[10], and $1/\gamma$ can be interpreted as the *cost effectiveness of R&D.* The bigger the cost effectiveness, the better. Since there are n symmetric firms with the same cost effectiveness parameter, n/γ is an indicator of sectoral cost effectiveness. After the steps given in the Appendix A.2, we can get the R&D spending of the firm given the R&D spending of the other firms. In other words, the reaction function for the R&D expenditure is

$$I_i = \frac{2[n-\alpha(n-1)][a-t-c+(2\alpha-1)\sum_{k\neq i}^{n} I_k]}{(1-s)b(n+1)^2\gamma - 2[n-\alpha(n-1)]^2} \tag{14}$$

Proposition 1 *i) If $\alpha > 0.5$, R&D investments are strategic complements. ii) If $\alpha < 0.5$, R&D investments are strategic substitutes.*

The proof is obvious. Therefore, if there is high technological transmission (*meaning* $\alpha > 0.5$*; e.g., there is no strong protection on intellectual property rights*) within the industry, firms enjoy the R&D investments of the other firms. In other words, the R&D investments of the other firms are supporting the equilibrium level of R&D investment of the firm in consideration. If there are low spillovers (*meaning* $\alpha < 0.5$), the technical information diffuses less. Then, the R&D investment of the other firms have negative effects on the firm's strategic choice of R&D investments[11]. Since the firms are all the same, the stage Nash equilibrium of the second stage of the game is given by

$$I_i^* = \frac{2[n-\alpha(n-1)](a-t-c)}{(1-s)b(n+1)^2\gamma - 2[n-\alpha(n-1)][1+\alpha(n-1)]} \tag{15}$$

This is the locally stable symmetric R&D profile. Here, the denominator is positive because of a local stability condition[12]. One immediate implication of (15) is that whether the equilibrium level of R&D in the regulated oligopoly is less or greater than its unregulated market equilibrium (*meaning* $t = 0$ *and* $s = 0$) depends *mainly* on the number of firms in

[10] This specification comes from Cheng (1984).
[11] The concept of strategic substitutes and complements comes originally from Bulow et al. (1985).
[12] A short discussion on the derivation of the stability condition is given in the Appendix A.8.

the industry.

4 Welfare Implications

In this section, we present the welfare implications not only of the decentralized equilibrium but also of a direct optimum in which a benevolent social planner can plan both the production and R&D levels directly. We show that, with the two policy tools, the decentralized equilibrium achieves the first-best equilibrium levels of output and R&D of the direct optimum. Therefore, in our model, the government's policy is nothing but a strategic commitment to ensure that the decentralized equilibrium achieves the social optimum.

4.1 Decentralized Equilibrium

In the decentralized case, the government can change the equilibrium levels indirectly via its tax-subsidy policies. Assume that the government maximizes a social welfare function of the standard type in which the tax revenue is redistributed[13].

$$W(t,s) = u(Q) - \frac{\gamma}{2}\sum_{k=1}^{n} I_k^2 - \sum_{k=1}^{n} q_k \Phi_k \tag{16}$$

So, the social welfare function is the sum of the surplus of the representative consumer and industry profits (*which is the total revenue from output sold minus the total cost of R&D and total cost of production*). After substituting for the utility function and imposing symmetry across the firms, social welfare is

$$W(t,s) = anq_i - b\frac{n^2 q_i^2}{2} - \frac{\gamma}{2} n I_i^2 - nq_i\Phi_i \tag{17}$$

[13]Remember we have an additively separable, money metric utility function.

The maximization of (17) gives us the following optimal policy mix for the government.

$$(t^*, s^*) = \left(\frac{b\gamma(a-c)}{[1+(n-1)\alpha]^2 - bn\gamma}, \frac{3+n}{1+n} - \frac{2}{1+\alpha(n-1)} \right) \tag{18}$$

The optimal tax/subsidy for R&D equates the marginal social return to R&D to its marginal cost, and the marginal social return to output to its marginal cost. Therefore, a committed government would announce the (t^*, s^*) social policy scheme. It is worth mentioning that the optimal subsidy to R&D investment is independent of a, b, c, γ. That is, the optimal subsidy is independent of demand characteristics, the initial marginal cost, and the cost effectiveness of R&D in our model. It depends only on the spillover parameter and number of firms in the industry. As can be seen from (18), given the number of firms in the sector, the degree of spillovers dictates the degree of government intervention via tax and subsidy policies.

By substituting the optimal policy values into the social welfare function we get

$$W^* = \frac{1}{2} \left(\frac{n\gamma(a-c)^2}{bn\gamma - [1+(n-1)\alpha]^2} \right) \tag{19}$$

Proposition 2 *The government subsidizes R&D investment iff the sector spillover is greater than* $\frac{1}{3+n}$. *If the sector spillover is less than* $\frac{1}{3+n}$, *taxing R&D investment is the optimum.*

The proof is given in the Appendix A.4. Subsidizing R&D is the known story. As was mentioned before, the government's motivation for subsidizing R&D is to make use of the positive externality of the spillovers. If they are sufficiently high, it is optimal for firms to under invest in R&D due to the very well known externality story. However, another interesting result emerges from the model. If the spillover is sufficiently low, given a fixed number of firms in the industry, the government might want to tax R&D in the decentralized equilibrium[14]. The intuition is the following. We have two effects in act in

[14] Even in a second best world in which output cannot be subsidized an R&D tax result still persists. A brief discussion of the second-best is given in the Appendix A.9.

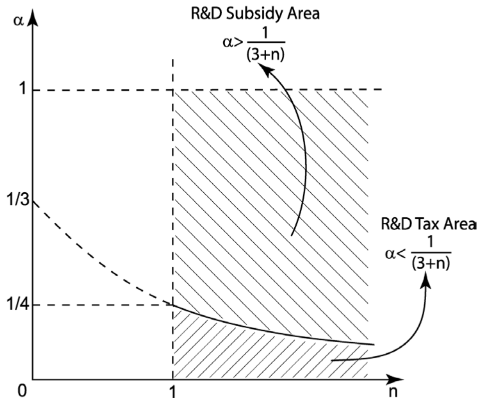
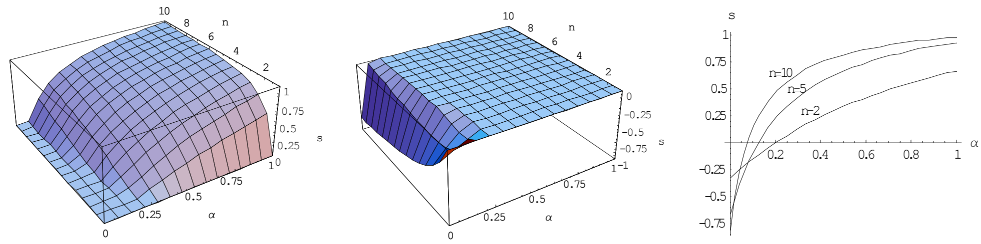


Figure 1: Optimal Subsidy and Tax Subspaces

the model, *namely a strategic effect and a spillover effect*. Strategic effect comes from the nature of the model. We have a Cournot model in which firms act strategically. They always want to increase their competitive advantage over the others. One way of doing that is to over invest in R&D for every given level of output. From the perspective of the society, that is choosing a higher research intensity than the desired level. To be able to focus only on this strategic effect assume for the moment that there are no spillovers. In that case, as shown in Figure 1, it is optimal for government to tax R&D investment for any number of firms in the industry. The motivation for a firm is to decrease its production cost more than any other firm by investing more in R&D and therefore to increase its competitiveness. This ends up with an undesirably higher level of research intensity for the society[15]. This is very similar to the defense strategies of two hostile countries such as India and Pakistan. It is individually rational for both countries to over invest in military forces. One country's resources devoted to defense makes it a greater danger for the other which creates incentives for over investment in army. This is, nevertheless, not a collectively rational behavior. We also have the spillover effect acting in the other way. The more

[15]Previous literature on R&D investment games tend to focus only on the equilibrium levels of R&D investment and output. We believe this might be misleading because of the following reason. As will be shown in Figure 3, in an R&D tax case equilibrium levels of both R&D investment and output can either be higher or lower than those of the social optimum whereas research intensity is always higher in the decentralized equilibrium. A tax on R&D creates disincentives for R&D investments for every given level of output for sure, but this does not necessarily end up with a lower level of equilibrium values.

R&D a single firm is engaged in, the more the sector benefits via spillover effects. In the absence of public policies, spillover effect decreases incentives for investing more in R&D intensive technologies because of the externalities involved. If the spillovers are sufficiently low, strategic effect dominates the spillover effect and as a result firms over invests in R&D for every given level of output. However, if they are high enough, spillover effect dictates the outcome and as a result firms underinvest in R&D for every given level of output.



(a)
3D Plot for optimal subsidy on R&D against spillover and the number of firms

(b)
3D Plot for optimal tax on R&D against spillover and the number of firms

(c)
Contour plot of optimal R&D tax/subsidy against spillover when the number of firms is 2, 5 or 10

Figure 2: Optimal Subsidy Plots

Figure 2 indicates that if the number of firms increases in the market, the taxation of R&D is optimum only with very low spillover levels. However, it is plausible to assume that spillovers in many industries cannot be very low and, for sure, zero (See, for example, Mansfield, 1985, 1992). Combining this evidence with Figure 1 and Figure 2, we conclude that an R&D tax is more likely to happen when there are a few firms in the industry. High performance of the market occurs when we have optimal subsidies associated with high spillovers or when we have optimal taxes associated with low spillovers.

The model we are providing here is a very simple one; consisting of linear demand functions and quadratic cost for R&D investments. This does not change the intuition, however. Even if a general demand function is used, an R&D tax result still persists since a linear demand function is always going to be a subset of a general demand function. The focus of the reader, therefore, should not be the optimal values of subsidies and taxes in this model but the existence of an R&D tax result even in a very general framework. One might argue that the model presented here is not suitable for the R&D games in consideration. Then, one should also criticize quite a large number of papers using the same D'Aspremont-Jacquemin framework to model R&D games.

Proposition 3 *The government subsidizes output* $\forall\alpha \in [0,1]$.

The proof is in Appendix A.5 and it directly follows from the comparison of the decentralized equilibrium with the socially desirable level of output. Output is subsidized in our model to offset the underproduction due to the market power. Since firms are acting in an oligopolistic market, they have market power, which lowers social welfare both at the margin and in the aggregate. Firms are producing too little output and this is bad for the well-being of the representative consumer. By subsidizing output, the government not only makes the firms better off, or at least no worse off, but it also increases the output provided to the consumer, both of which generate a welfare improvement. It is, indeed, always socially optimal to give a subsidy to output to close the gap between the marginal cost of production and the price of output.

The subgame perfect Nash equilibrium of the game is the following R&D and output profiles

$$I_i^* = \frac{(a-c)[1+(n-1)\alpha]}{bn\gamma - [1+(n-1)\alpha]^2} \tag{20}$$

$$q_i^* = \frac{(a-c)\gamma}{bn\gamma - [1+(n-1)\alpha]^2} \tag{21}$$

where the second order condition requires $bn\gamma - [1+(n-1)\alpha]^2 > 0$.

Proposition 4 *The government's optimal policy scheme* (t^*, s^*) *works if the number of firms in the market is less than* $b\gamma$.

The proof is in the Appendix A.6. In mathematical terms, Proposition 4 says that $n < b\gamma$ where $1/\gamma$ is the cost effectiveness of one firm, and b is the output sensitivity of demand

$(dp(Q)/dQ)$. The right hand side of this inequality depends on parameters which cannot be changed immediately. Demand characteristics remain the same unless there is a shock, and so does the cost effectiveness of R&D. Number of firms in the industry has to be less than the multiplication of those two. Thus, the optimal policy scheme works only if the sector is reasonably concentrated.

When the government commits to an optimal policy scheme (t^*, s^*), it assumes that the number of firms in the market is fixed and will be no more than $b\gamma$. However, as long as there are positive profits, the number of firms in the industry increases. If the number of firms in the industry increases, the government should commit to another optimal policy scheme. Therefore, the optimal policy scheme (t^*, s^*) works if there are barriers to deter entry. This is one of the conventional assumptions in the policy games. However, we need more than that for a working policy. There has to be a ceiling for the number of firms in the industry which guarantees the inequality given in the Proposition 4 to hold. Otherwise the optimal strategy for the firms is not to invest in R&D and not to produce. The need for a ceiling for the number of firms is also consistent with Spence (1984). He finds that incentives for R&D rise and then fall as concentration declines.

4.2 Direct Optimum[16]

As a benchmark, consider the direct optimum, the optimum of a welfare maximizing social planner. That is, assume there is a benevolent social planner who has direct control over both the output produced and the R&D investment. The social welfare function of the planner takes the same form as before, but since he has direct control over output and R&D, his choice variables are now the R&D investment profile $\boldsymbol{I} = (I_1, I_2, ..., I_n)$, and the output profile $\boldsymbol{q} = (q_1, q_2, ..., q_n)$.

$$\hat{W}(\boldsymbol{q}, \boldsymbol{I}) = u(Q) - \frac{\gamma}{2}\sum_{k=1}^{n} I_k^2 - \sum_{k=1}^{n} q_k \Phi_k \tag{22}$$

[16] Some details are provided in Appendix A.7.

Therefore,

$$\hat{W}(\boldsymbol{q}, \boldsymbol{I}) = a\sum_{k=1}^{n} q_k - b\frac{(\sum_{k=1}^{n} q_k)^2}{2} - \frac{\gamma}{2}\sum_{k=1}^{n} I_k^2 - \sum_{k=1}^{n} q_k \Phi_k \tag{23}$$

There is still some possibility for imperfect spillovers due to idiosyncratic effects, imperfect communication, and so forth. The first orders conditions for the social planner are

$$\frac{\partial \hat{W}(\boldsymbol{q}, \boldsymbol{I})}{\partial q_i} = a - b\sum_{k=1}^{n} q_k - \Phi_i = 0 \tag{24}$$

and

$$\frac{\partial \hat{W}(\boldsymbol{q}, \boldsymbol{I})}{\partial I_i} = -\gamma I_i + q_i + \alpha \sum_{k \neq i}^{n} q_k = 0 \tag{25}$$

The FOC with respect to R&D investment indicates that the social marginal cost of R&D investment has to equal to the total output from the firms resulting from the marginal R&D investment. After some manipulation the output and R&D levels chosen by the social planner are:

$$I_i^S = \frac{(a-c)[1+(n-1)\alpha]}{bn\gamma - [1+(n-1)\alpha]^2} \tag{26}$$

$$q_i^S = \frac{(a-c)\gamma}{bn\gamma - [1+(n-1)\alpha]^2} \tag{27}$$

where $b\gamma > n$. These are the same equilibria we get from the decentralized economy with the government intervention via tax-subsidy policies. Therefore, the decentralized economy achieves the first-best optimum.

4.3 Comparison of the Direct Optimum with the Market Equilibrium

A comparison of the FOCs of the direct optimum with the FOCs of the market equilibrium gives more insight on how the mechanism works. We define the market equilibrium as the outcome in which there is no government intervention via tax and subsidy policies. In an unregulated market R&D incentives are deteriorated by the existence of spillovers. The

FOCs for the market equilibrium are as follows.

$$2bq_i + b\sum_{k\neq i}^{n} q_k = a - \Phi_i \tag{28}$$

$$2b\frac{\partial q_i}{\partial I_i} q_i = \gamma I_i \tag{29}$$

where $\frac{\partial q_i}{\partial I_i} = \frac{n-\alpha(n-1)}{b(n+1)}$. From (28), which is the FOC with respect to output, we get

$$b(n+1)q_i = (a-c) + [1+\alpha(n-1)]I_i \tag{30}$$

From (29), which is the FOC with respect to R&D investment,

$$2[n-\alpha(n-1)]q_i = (n+1)\gamma I_i \tag{31}$$

(30) and (31) characterizes the market equilibrium in $I-q$ space. The FOCs of the direct optimum are

$$nbq_i = (a-c) + [1+\alpha(n-1)]I_i \tag{32}$$

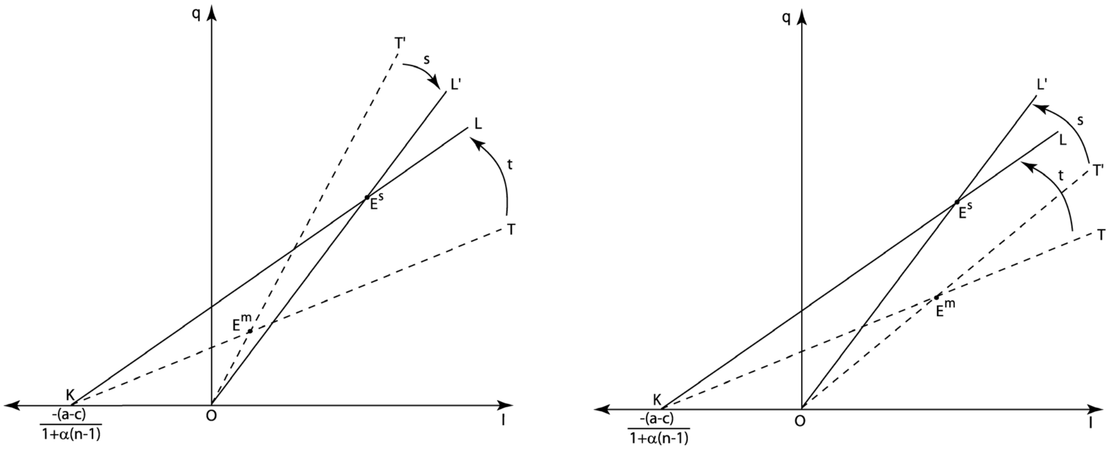
$$[1+\alpha(n-1)]q_i = \gamma I_i \tag{33}$$

Equation (32), which is the FOC with respect to output, and equation (33), which is the FOC with respect to R&D investment, characterize the direct optimum in the $I-q$ space. It is worth mentioning that equations (31) and (33) define the research intensity in the market equilibrium and the direct optimum, respectively.

Market Equilibrium	$b(n+1)q_i = (a-c) + [1+\alpha(n-1)]I_i$ $2[n-\alpha(n-1)]q_i = (n+1)\gamma I_i$
Direct Optimum	$nbq_i = (a-c) + [1+\alpha(n-1)]I_i$ $[1+\alpha(n-1)]q_i = \gamma I_i$

Table 1: First Order Conditions

The FOCs for the market equilibrium and direct optimum are summarized in Table 1. By making use of these two pairs of FOCs one can get the optimal tax/subsidy policy which we derived before. More insight can be gotten from the graphs of these equations in the $I - q$ space. Graphs of equations (30), (31), (32) and (33) are given in Figure 3[17]. Market equilibrium is characterized by lines KT and OT', and the social optimum is characterized by lines KL and OL'. The slope of KL is always greater than the slope of KT. That means the optimal policy towards market power is always to subsidize output (Proposition 3). However, the slope of OL' is smaller than the slope of OT' only if $\alpha > 1/(3+n)$. In this situation, it is socially optimal to subsidize R&D as shown in Figure 3a. However, if $\alpha > 1/(3+n)$, the slope of OL' is greater than the slope of OT', meaning that taxing R&D investment is the optimum (Proposition 2). This situation is shown in Figure 3b.



(a) R&D Subsidy:
Market Equilibrium and Social Optimum
when $\alpha > 1/(3+n)$

(b) R&D Tax:
Market Equilibrium and Social Optimum
when $\alpha < 1/(3+n)$

Figure 3: Characterization of the Social Policy

As mentioned before, when a firm engages in R&D investment, it is affected twice. First, the production cost decreases due to the *cost reducing behavior* of R&D investments. Second, the production cost of the other firms decreases due to the spillover effects, which in turn

[17] Figure 3b illustrates the case in which $\frac{n(n-2)-1}{(n-1)[n(n+4)+1]} < \alpha < \frac{1}{3+n}$. That assures both R&D and output profiles of the social optimum are bigger than those of the market equilibrium ($I_i^S > I_i^M$ and $q_i^S > q_i^M$). However, if $\alpha < \frac{n(n-2)-1}{(n-1)[n(n+4)+1]}$ then $I_i^S < I_i^M$ and $q_i^S < q_i^M$. One important point is that the equilibrium level of both R&D investment and output are greater in the social optimum than in the market equilibrium in both of the cases shown in the Figure 3. That is, the market fails to provide the optimal amount of output and R&D investment in the absence of a regulation.

decreases the competitive advantage of the firm. This second effect is the source of the strategic effect. Therefore, the investment decision in R&D depends purely on which effect dominates the others. Whenever the spillover effects are high enough, firms collectively choose too low a level of R&D conditional on output compared to the social optimum. Then the system is characterized by Figure 3a and the government should impose an R&D subsidy to hit the first-best frontier. However, if the spillovers are tiny, it is in firms' best interest to choose collectively too high a level of R&D conditional on output. In this case, spillover effect is tiny and we essentially have the strategic effect. Thus, firms are mostly dependent on their own level of R&D investments and do not have much incentive to free-ride. This results in an over investment in R&D conditional on output level. Such a situation is shown in Figure 3b. In both of the cases either an R&D subsidy or an R&D tax carries the industry to its social optimum with the right research intensity.

An important point is that we can reach to the first best optimum since we have two policies, *namely output subsidies and R&D subsidies* (*or taxes*), addressing the two sources of problems in the industry, *namely market power and the incentive reducing behavior of spillovers*. Therefore, a committed government can achieve Pareto optimality within the standard decentralized competitive market simply by levying proper taxes or subsidies that direct competitive behavior to the correct Pareto optimal conditions.

5 Conclusion

In this paper we build up a simple framework which is quite flexible and useful for analyzing many kinds of public policies towards R&D investments. R&D tax credits can reduce the cost of industrial innovation by providing a rebate through the tax system. However, the controversy over the effectiveness of R&D tax incentives is still considerable. Some empirical studies claimed that R&D tax incentives did not work well. Some others find that R&D tax incentives are effective.

R&D is essentially a public good. If it is priced as though it were a private good, the outcome will be inefficient. There is a well-known externality associated with R&D investment coming through spillover effects which are a source of market failure. Such failures generally cause firms to underinvest in research. On the other hand, our analysis shows that in an oligopolistic market firms might want to over invest in R&D due to strategic reasons. Therefore, the market will fail to provide the right amount of R&D. The most direct way to correct this problem is to subsidize (*or tax*) the R&D investment.

We set up a simple game-theoretical model to identify the conditions under which tax and subsidy policies serve as the means to reach the socially desirable level of privately-financed R&D spending in a regulated oligopolistic industry. It is shown that, in the decentralized equilibrium, the government might want to tax R&D. Here the tax/subsidy policy changes the vector of relative prices in the economy from their values in the market equilibrium to the socially optimal ones so as to reach the Pareto efficiency. However, this is more likely to happen when there are a few firms in the industry. We also show that governments tax/subsidy policy works only if the industry is reasonably concentrated. High concentration ratios are guaranteed by the existence of high R&D investment costs.

We present results both graphically and analytically which helped us to give more insights on the mechanism. When a tax-subsidy policy is considered, we show that we should focus on not only the equilibrium level of R&D or output but also the research intensity. For example, a tax on R&D lowers the level of R&D investment for every given level of output which would not necessarily end up with a lower or higher level of investment per se.

In our model, optimal R&D subsidy is independent of demand characteristics, the initial marginal cost, and the cost effectiveness of R&D. It depends only on the R&D spillovers and the number of firms in the industry. Therefore, given the number of firms in an industry, the government's intervention is a function of the degree of spillovers.

The reader should keep in mind that our analysis is a partial equilibrium analysis. We neglect the general equilibrium interactions in the economy when the government uses tax/subsidy policies. Therefore, care should be taken in interpreting the conclusions derived from the model. Our aim with this partial equilibrium analysis is to give some insight into the characteristics of R&D tax incentives in oligopolies. One might argue that giving an output subsidy to the oligopolistic firms seems odd since it is increasing their profits but these profits, too, are taxed in the real life. Governments tax firms commonly to raise general revenue. They use some of this to finance public infrastructure and to enforce contracts via courts. . . etc., which help firms. Thus, a subsidy in a partial equilibrium model can be viewed as a lower tax in a general equilibrium context. One analogy can be the existence of both export subsidies and tariffs.

When we consider tax incentives in the real life one practical problem emerges immediately. We assume that the government can clearly define what constitutes an investment in R&D, and that the firms do not misreport their R&D investments. In reality, this is not the case. Governments are not able to define R&D activities perfectly, and firms have an incentive to report some other activities as if they were R&D expenditures. Both have been important problems in practice. Moreover, in an ideal world, the government should determine the sectors that have the highest social rate of return after an extensive cost-benefit analysis. In reality, there is a tendency to reward lobbyists and other influential groups instead of decisions on the basis of cost-benefit analysis.

A Appendix

A.1 Third stage

Firm i's profit is given by

$$\Pi_i(q_i, \boldsymbol{q}_{-i}, I_i, \boldsymbol{I}_{-i}) = [p(q_i, \boldsymbol{q}_i) - t]q_i - q_i\Phi_i - (1-s)\Psi(I_i) \qquad \forall i \in \{1, 2, ..., n\} \tag{7}$$

Therefore, the FOC is

$$\frac{\partial \Pi_i}{\partial q_i} = p(q_i, \boldsymbol{q}_{-i}) - t - \Phi_i + \frac{\partial p(q_i, \boldsymbol{q}_{-i})}{\partial q_i} q_i = 0 \tag{A-1}$$

With a linear demand function of the form $p(q_i, \boldsymbol{q}_{-i}) = a - b\sum_{k=1}^{n} q_k$, we have

$$q_i = \frac{a - b\sum_{k\neq i}^{n} q_k - \Phi_i - t}{2b} \qquad \forall i \in \{1, 2, ..., n\} \tag{8}$$

Thus

$$\begin{aligned}
2bq_1 &= a - b(0 + q_2 + q_3 + ... + q_n) - \Phi_1 - t \\
2bq_2 &= a - b(q_1 + 0 + q_3 + ... + q_n) - \Phi_2 - t \\
&\vdots \qquad\qquad\qquad \vdots \qquad\qquad\qquad \vdots \\
2bq_n &= a - b(q_1 + q_2 + ... + q_{n-1} + 0) - \Phi_n - t
\end{aligned}$$

These equations form a system of n equations in n unknowns. Therefore,

$$2b\sum_{k=1}^{n} q_k = na - b(n-1)\sum_{k=1}^{n} q_k - \sum_{k=1}^{n} \Phi_k - nt \tag{A-2}$$

$$\iff \sum_{k=1}^{n} q_k = \frac{n(a-t) - \sum_{k=1}^{n} \Phi_k}{b(n+1)} \tag{A-3}$$

For future reference note that this can be rearranged to get

$$\sum_{k\neq i}^{n} q_k = \frac{n(a-t) - \sum_{k=1}^{n} \Phi_k}{b(n+1)} - q_i \tag{A-4}$$

$$\implies q_i = \frac{a - b\frac{n(a-t) - \sum_{k=1}^{n} \Phi_k}{b(n+1)} + bq_i - \Phi_i - t}{2b} \tag{A-5}$$

$$\implies q_i = \frac{a - t - (n+1)\Phi_i + \sum_{k=1}^{n} \Phi_k}{b(n+1)} \tag{9}$$

A.2 Second Stage

$$\begin{aligned}
\Pi_i(q_i, \boldsymbol{q}_{-i}, I_i, \boldsymbol{I}_{-i}) &= [p(q_i, \boldsymbol{q}_{-i}) - t]q_i - q_i\Phi_i - (1-s)\Psi(I_i) \qquad \forall i \in \{1, 2, ..., n\} \qquad (7) \\
\Pi_i(q_i, \boldsymbol{q}_{-i}, I_i, \boldsymbol{I}_{-i}) &= [a - b\sum_{k=1}^{n} q_k - t - \Phi_i]q_i - (1-s)\Psi(I_i)
\end{aligned}$$

By substituting for $\sum_{k=1}^{n} q_k$ and q_i

$$\hat{\Pi}_i(I_i, \boldsymbol{I}_{-i}) = [a - b\left(\frac{n(a-t) - \sum_{k=1}^{n}\Phi_k}{b(n+1)}\right) - t - \Phi_i]\left(\frac{a - t - (n+1)\Phi_i + \sum_{k=1}^{n}\Phi_k}{b(n+1)}\right) - (1-s)\Psi(I_i)$$

$$\implies \hat{\Pi}_i(I_i, \boldsymbol{I}_{-i}) = b\left(\frac{a - t - (n+1)\Phi_i + \sum_{k=1}^{n}\Phi_k}{b(n+1)}\right)^2 - (1-s)\Psi(I_i)$$

which boils down to

$$\hat{\Pi}_i(q_i(I_i, \boldsymbol{I}_{-i}), I_i) = bq_i^2 - (1-s)\Psi(I_i) \tag{10}$$

First order conditions for the second stage of the game is then,

$$\frac{\partial \hat{\Pi}_i}{\partial I_i} = 2b\frac{\partial q_i}{\partial I_i}q_i - (1-s)\Psi'(I_i) = 0 \qquad \forall i \in \{1, 2, ..., n\} \tag{11}$$

The marginal cost of R&D is $(1-s)\Psi'(I_i)$, which is equal to marginal benefit $2b\frac{\partial q_i}{\partial I_i}q_i$. Moreover,

$$\begin{aligned} \Phi_i(I_i, \boldsymbol{I}_{-i}) &= c - [(1-\alpha)I_i + \alpha\sum_{k=1}^{n} I_k] \\ &= c - (1-\alpha)I_i - \alpha(I_1 + I_2 + ... + I_n) && \text{(A-7)} \\ \sum_{k=1}^{n}\Phi_k &= nc - (1-\alpha)\sum_{k=1}^{n} I_k - n\alpha\sum_{k=1}^{n} I_k \\ &= nc - [1 + \alpha(n-1)]\sum_{k=1}^{n} I_k && \text{(A-8)} \end{aligned}$$

Hence,

$$(1-s)\Psi'(I_i) = 2\frac{\partial q_i}{\partial I_i}\left(\frac{a - t - (n+1)\Phi_i + \sum_{k=1}^{n}\Phi_k}{n+1}\right) \tag{A-9}$$

By using $\frac{\partial q_i}{\partial I_i} = \frac{(n+1)-1-\alpha(n-1)}{b(n+1)}$

$$(1-s)\Psi'(I_i) = 2\left(\frac{(n+1) - 1 - \alpha(n-1)}{b(n+1)}\right)\left(\frac{a - t - (n+1)\Phi_i + \sum_{k=1}^{n}\Phi_k}{n+1}\right) \tag{A-10}$$

Define the cost of R&D investment as

$$\Psi(I_i) = \gamma\frac{I_i^2}{2} \tag{13}$$

After some rearrangement, the reaction function in terms of R&D investments for firm i is given by

$$I_i = \frac{2[n - \alpha(n-1)][a - t - c + (2\alpha - 1)\sum_{k\neq i}^{n} I_k}{(1-s)b(n+1)^2\gamma - 2[n - \alpha(n-1)]^2} \tag{14}$$

Here, the denominator is positive by local stability condition. Since the firms are all the same the stage Nash equilibrium of the second stage of the game is given by

$$I_i^* = \frac{2[n - \alpha(n-1)](a - t - c)}{(1-s)b(n+1)^2\gamma - 2[n - \alpha(n-1)][1 + \alpha(n-1)]} \tag{15}$$

A.3 Derivation of Observation 1:

From (11) we know that $(1-s)\Psi'(I_i) = 2b\frac{\partial q_i}{\partial I_i}q_i$. Here, the differentiation of output with respect to R&D investment can be decomposed as follows:

$$\frac{\partial q_i}{\partial I_i} = \frac{\frac{\partial(\sum_{k=1}^{n}\Phi_k)}{\partial I_i}}{b(n+1)} - \frac{\frac{\partial \Phi_i}{\partial I_i}}{b} \tag{A-11}$$

Hence,

$$\begin{aligned}(1-s)\Psi'(I_i) &= 2bq_i\left(\frac{\frac{\partial(\sum_{k=1}^{n}\Phi_k)}{\partial I_i}}{b(n+1)} - \frac{\frac{\partial \Phi_i}{\partial I_i}}{b}\right) \\ &= \frac{2q_i}{n+1}\left(\sum_{k=1}^{n}\left(\frac{\partial \Phi_k}{\partial I_i} - \frac{\partial \Phi_i}{\partial I_i}\right) - \frac{\partial \Phi_i}{\partial I_i}\right)\end{aligned} \tag{A-12}$$

This can also be written in terms of elasticities. Let $\frac{\partial \Phi_i}{\partial I_j}$ as the sensitivity of marginal cost of $firm$ i with respect to R&D expenditure of $firm\ j$. Therefore, elasticity of marginal cost of $firm\ i$ with respect to the R&D investment of $firm\ j$ is defined as

$$\eta_{\Phi_i:I_j} = \frac{I_j}{\Phi_i}\frac{\partial \Phi_i}{\partial I_j} \tag{A-13}$$

Thus,

$$(1-s)\Psi'(I_i) = \underbrace{\frac{2}{(n+1)}}_{\text{mark-up}} \underbrace{\left(\overbrace{\sum_{k\neq i}^{n}(\eta_{\Phi_k:I_i}\Phi_k)}^{\text{inter-firm effect}} - n\overbrace{(\eta_{\Phi_i:I_i}\Phi_i)}^{\text{own effect}}\right)}_{\text{total technology change}} \underbrace{\frac{q_i}{I_i}}_{\text{inverse of research intensity}} \tag{12}$$

A.4 Proof of Proposition 2:

We have already found that $s = \frac{3+n}{1+n} - \frac{2}{1+\alpha(n-1)}$. If $s > 0$, then $\frac{3+n}{1+n} - \frac{2}{1+\alpha(n-1)} > 0$. With n greater than or equal to 1 this reduces $\alpha > \frac{1}{3+n}$.

A.5 Proof of Proposition 3:

Simple comparison of the decentralized equilibrium's FOC's with the direct optimum FOC's shows that $t^* = -bq_i^*$ where $q_i^* > 0$. Therefore, the government subsidizes output.

A.6 Proof of Proposition 4:

By looking at the signs of the leading principal minors it is easy to show that SOC requires $bn\gamma - [1+(n-1)\alpha]^2 > 0$. In the worst case $\alpha = 1$. When it is the case

$$\begin{aligned} bn\gamma - n^2 &> 0 \\ &\iff n(b\gamma - n) > 0 \\ &\iff n < b\gamma \end{aligned} \tag{A-14}$$

A.7 Direct Optimum

$$\hat{W}(\boldsymbol{q}, \boldsymbol{I}) = u(Q) - \frac{\gamma}{2}\sum_{k=1}^{n} I_k^2 - \sum_{k=1}^{n} q_k \Phi_k \tag{22}$$

$$\hat{W}(\boldsymbol{q}, \boldsymbol{I}) = a\sum_{k=1}^{n} q_k - b\frac{(\sum_{k=1}^{n} q_k)^2}{2} - \frac{\gamma}{2}\sum_{k=1}^{n} I_k^2 - \sum_{k=1}^{n} q_k \Phi_k \tag{23}$$

The marginal cost function is $\Phi_i(I_i, \boldsymbol{I}_{-i}) = c - (1-\alpha)I_i - \alpha\sum_{k=1}^{n} I_k$. The first order conditions for the social planner are

$$\frac{\partial \hat{W}(\boldsymbol{q}, \boldsymbol{I})}{\partial q_i} = a - b\sum_{k=1}^{n} q_k - \Phi_i = 0 \tag{24}$$

$$\frac{\partial \hat{W}(\boldsymbol{q}, \boldsymbol{I})}{\partial I_i} = -\gamma I_i + q_i + \alpha\sum_{k\neq i}^{n} q_k = 0 \tag{25}$$

From (24) $a - b\sum_{k=1}^{n} q_k - c + (1-\alpha)I_i + \alpha\sum_{k=1}^{n} I_k = 0$. From (25) $[1+\alpha(n-1)]\sum_{k=1}^{n} q_k = \gamma\sum_{k=1}^{n} I_k$. Solving both together we get

$$na - nb\sum_{k=1}^{n} q_k = nc - \frac{[1+\alpha(n-1)]^2}{\gamma}\sum_{k=1}^{n} q_k \tag{A-15}$$

By imposing symmetry $I_i^S = \frac{(a-c)[1+(n-1)\alpha]}{bn\gamma - [1+(n-1)\alpha]^2}$ and $q_i^S = \frac{(a-c)\gamma}{bn\gamma - [1+(n-1)\alpha]^2}$ where $b\gamma > n$.

A.8 Stability Condition

Here, we do not go into details of the stability discussion in Cournot markets, since it is out of the scope of this paper[18]. Instead, we will derive the local stability condition that we need.

The stage Nash equilibrium of the game is locally stable iff

$$\frac{\partial^2 \hat{\Pi}_i^*}{\partial I_i^2} + (n-1)\frac{\partial^2 \hat{\Pi}_i^*}{\partial I_i \partial I_j} < 0 \qquad \forall n; \quad i \neq j; \quad i, j \in \{1, 2, ...n\} \tag{A-16}$$

[18] For a discussion of stability in oligopolies see Fisher (1961), Furth (1986), Hahn (1962), McManus and Quandt (1961), Teocharis (1960).

where $\hat{\Pi}_i^*$ represents the equilibrium level of profits and

$$\frac{\partial^2 \hat{\Pi}_i^*}{\partial I_i^2} = 2b\left(\left(\frac{\partial q_i}{\partial I_i}\right)^2 + q_i \frac{\partial^2 q_i}{\partial I_i^2}\right) - (1-s)\gamma \tag{A-17a}$$

$$= \frac{2[n-\alpha(n-1)]^2}{b(n+1)^2} - (1-s)\gamma \tag{A-17b}$$

$$\frac{\partial^2 \hat{\Pi}_i^*}{\partial I_i \partial I_j} = 2b\left(\frac{\partial^2 q_i}{\partial I_i \partial I_j} q_i + \frac{\partial q_i}{\partial I_i}\frac{\partial q_i}{\partial I_j}\right) \tag{A-18a}$$

$$= \frac{2[n-\alpha(n-1)](2\alpha-1)}{b(n+1)^2} \tag{A-18b}$$

Therefore, the stability condition is

$$(1-s)b(n+1)^2\gamma - 2[n-\alpha(n-1)][1+\alpha(n-1)] > 0 \tag{A-19}$$

which is exactly the same as the denominator of the equilibrium level of R&D spending.

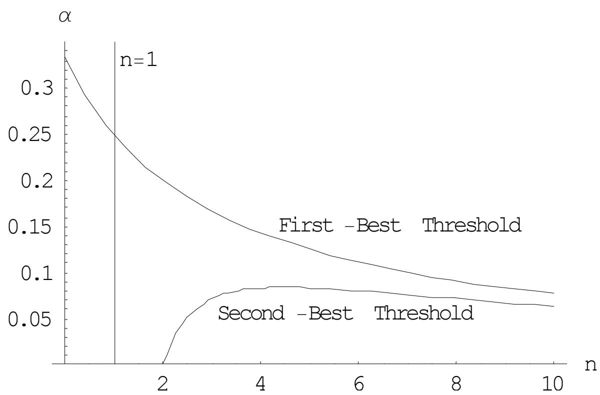


Figure 4: First-best and Second-best Thresholds

A.9 Second-Best Optimum

In this case we cannot reach the social optimum, but R&D tax result is still preserved. Suppose it is not possible to subsidize output. This can be obtained from our calculations for the first-best simply by setting $t = 0$ in the social welfare function. Then, the optimal subsidy in the second-best is as follows:

$$s^{SB} = \frac{2-n+(n-1)(n+4)\alpha}{2+n+(n+2)(n-1)\alpha} \tag{A-20}$$

where SB stands for second-best. It is optimal to tax R&D whenever $\alpha < \frac{n-2}{(n-1)(n+4)}$. Figure 4 shows the first-best and second-best thresholds at the same graph. First-best threshold is the same curve given in Figure 1 and the second-best threshold is the counterpart of it in the second best environment. In a second-best environment, taxing R&D is optimal anywhere below that threshold.

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