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Endogenously persistent output dynamics: A puzzle for the sticky-price model?*

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Comments welcome

Abstract

I show that endogenously persistent output dynamics are not a puzzle for the standard sticky-price model once openness of the economy is taken into account. I make this point using a two-country, monetary model of macroeconomic interdependence under internationally incomplete asset markets with stationary net foreign asset dynamics. If asset markets are incomplete, price stickiness generates endogenous persistence in the cross-country GDP differential by introducing persistence in the dynamics of the relative price differential between the two economies. This results in the dependence of the current real GDP differential on its past value, as well as on the stock of net foreign assets accumulated in the previous period. The elasticity of the current GDP differential to its past value is sizable for standard parameter values, implying a quantitatively significant persistence effect through this channel. Endogenous persistence yields hump-shaped responses of GDP to productivity and monetary policy shocks.

Keywords: Dynamics; Incomplete markets; Persistence; Open economy; Sticky prices
JEL Classification: E32; F41

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1 Introduction

A familiar result of the recent closed economy, macro literature is that sticky-price models based on the New Keynesian, forward looking Phillips curve cannot generate endogenous persistence in key macroeconomic variables, namely, GDP.¹ This is generally interpreted as a failure of the sticky-price benchmark to match key features of the data.

I show that endogenously persistent output dynamics are not a puzzle for the standard Calvo-Yun-Rotemberg, sticky-price model once openness of the economy is taken into account. I make this point using the two-country, monetary model of macroeconomic interdependence under internationally incomplete asset markets developed in Cavallo and Ghironi (2001). The model features stationary net foreign asset dynamics and endogenous interest rate setting as in Taylor (1993). Stationarity of net foreign assets is necessary to avoid permanent effects of temporary shocks via wealth redistribution across countries. Market incompleteness matters for endogenous persistence because persistence of individual country dynamics is generated through persistent dynamics of cross-country differentials, which disappear when asset markets are internationally complete (or the elasticity of substitution between domestic and foreign goods equals 1). Similarly, openness is crucial. If the home economy coincides with the world economy, domestic GDP coincides with world GDP regardless of asset market structure. If the economy is open and asset markets are incomplete, price stickiness generates endogenous persistence in the cross-country GDP differential (and in other real variables) by introducing persistence in the dynamics of the relative price differential between the two economies. This results in the dependence of the current real GDP differential on its past value, as well as on the stock of net foreign assets accumulated in the previous period. The elasticity of the current GDP differential to its past value is sizable for standard parameter values, implying a quantitatively significant persistence effect through this channel.²

Openness of the economy and market incompleteness amplify the response of GDP to productivity and monetary policy shocks.³ Christiano, Eichenbaum, and Evans (2001) generate endogenous persistence and non-monotone dynamics in a closed economy model by combining sticky prices and wages with habit persistence in preferences for consumption, adjustment costs in investment, and variable capital utilization. Papadopoulou (2001) argues that a combination of sticky prices, limited participation, and endogenous monetary policy can generate hump-shaped responses to

¹See Clarida, Galí, and Gertler (1999) and Chari, Kehoe, and McGrattan (2000), for example.

²The two-country, sticky-price model of this paper delivers quite different dynamics under complete and incomplete markets, reinforcing the conclusions of the flexible-price analysis of Ghironi (2000). This differs from the results of the two-country, flexible-price models in Heathcote and Perri (2002) and Kehoe and Perri (2002) and of the small open economy, flexible-price setup in Schmitt-Grohé and Uribe (2001).

³Razin and Yuen (2001) show that consumption smoothing under complete markets magnifies output responses to nominal GDP shocks in a small open economy.

shocks. Endogenous persistence via openness and market incompleteness generates hump-shaped responses of GDP (and other real variables) to productivity and monetary policy shocks in a model that does not include capital accumulation without the need for limited participation and/or habit persistence. Thus, if one believes that the world as a whole is the only economy that can be truly treated as closed, reasonable assumptions about openness of the economy and asset market structure can help explain some empirical observations that have been puzzling (closed economy) macro theorists in a standard sticky-price setup. Extending the model to include capital accumulation and adjustment costs in investment would reinforce the results.

The rest of the paper is as follows. Section 2 describes the setup of the model. Section 3 presents the log-linear system used to analyze dynamics. Section 4 briefly described world dynamics under sticky prices. Section 5 focuses on cross-country differences. Section 6 assesses the quantitative significance of the results. Section 7 concludes.

2 The model

The model follows Cavallo and Ghironi (2001). Readers who are familiar with that paper can skip this section and move on to Section 3.5.

The model relies on Weil's (1989) demographic structure to pin down steady-state net foreign assets and generate stationary dynamics under incomplete markets.⁴ The world consists of two countries, *home* and *foreign*. In each period t , the world economy is populated by a continuum of infinitely lived households between 0 and N_t^W . Each household consumes, supplies labor, and holds financial assets. Households are born on different dates *owning no assets*, but they own the present discounted value of their labor income.⁵ The number of households in the home economy, N_t , grows over time at the exogenous rate $n > 0$, *i.e.*, $N_{t+1} = (1+n)N_t$. I normalize the size of a household to 1, so that the number of households alive at each point in time is the economy's population. Foreign population (N_t^*) grows at the same rate as home population. The world economy has existed since the infinite past. It is useful to normalize world population at time 0 to the continuum between 0 and 1, so that $N_0^W = 1$.

A continuum of goods $i \in [0, 1]$ are produced in the world by monopolistically competitive, infinitely lived firms, each producing a single differentiated good. Firms have existed since the infinite past. At time 0, the number of goods that are supplied in the world economy is equal to the number of households. The latter grows over time, but the commodity space remains unchanged.

⁴See Ghironi (2000) and Schmitt-Grohé and Uribe (2001) for discussions of alternative approaches. Without stationary net foreign assets, temporary shocks would have permanent consequences as in Obstfeld and Rogoff (1995).

⁵Blanchard (1985) combines this assumption with a positive probability of not surviving until the next period. This is advantageous for calibration purposes (see below), besides being plausible. The Weil setup is simpler to illustrate.

Thus, as time goes, the ownership of firms spreads across a larger number of households. Profits are distributed to consumers via dividends, and the structure of the market for each good is taken as given. The domestic economy produces goods in the interval $[0, a]$, which is also the size of the home population at time 0, whereas the foreign economy produces goods in the range $(a, 1]$.

The asset menu includes nominal, uncontingent bonds denominated in units of domestic and foreign currency, money balances, and shares in firms. Private agents in both countries trade the bonds domestically and internationally. Shares in home (foreign) firms and domestic (foreign) currency balances are held only by home (foreign) residents.

2.1 Households

Agents have perfect foresight, though they can be surprised by initial unexpected shocks. Consumers have identical preferences over a real consumption index (C), labor effort supplied in a competitive market (L), and real money balances ($\frac{M}{P}$, where M denotes nominal money holdings and P is the consumption-based price index–CPI). I normalize the endowment of time in each period to 1. At any time t_0 , the representative home consumer j born in period $v \in [-\infty, t_0]$ maximizes the intertemporal utility function:

$$U_{t_0}^{v^j} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\rho \log C_t^{v^j} + (1 - \rho) \log (1 - L_t^{v^j}) + \chi \log \frac{M_t^{v^j}}{P_t} \right], \quad (1)$$

with $0 < \rho < 1$.⁶

The consumption index for the representative domestic consumer is a standard CES aggregator of foreign and domestic sub-indexes: $C_t^{v^j} = \left[a^{\frac{1}{\omega}} \left(C_{Ht}^{v^j} \right)^{\frac{\omega-1}{\omega}} + (1-a)^{\frac{1}{\omega}} \left(C_{Ft}^{v^j} \right)^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}}$, where $\omega > 0$ is the intratemporal elasticity of substitution between domestic and foreign goods. The consumption sub-indexes that aggregate individual domestic and foreign goods are $C_{Ht}^{v^j} = \left[\left(\frac{1}{a} \right)^{\frac{1}{\theta}} \int_0^a c_t^{v^j}(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$ and $C_{Ft}^{v^j} = \left[\left(\frac{1}{1-a} \right)^{\frac{1}{\theta}} \int_a^1 c_{*t}^{v^j}(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$, respectively, where $c_{*t}^{v^j}(i)$ denotes time t consumption of good i produced in the foreign country, and $\theta > 1$ is the elasticity of substitution between goods produced inside each country.

The CPI is $P_t = \left[a P_{Ht}^{1-\omega} + (1-a) P_{Ft}^{1-\omega} \right]^{\frac{1}{1-\omega}}$, where P_H (P_F) is the price sub-index for home (foreign)-produced goods—both expressed in units of the home currency. Letting $p_t(i)$ be the home currency price of good i , we have $P_{Ht} = \left(\frac{1}{a} \int_0^a p_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}$ and $P_{Ft} = \left(\frac{1}{1-a} \int_a^1 p_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}$.

I assume that there are no impediments to trade and that firms do not engage in local currency pricing (*i.e.*, pricing in the currency of the economy where goods are sold). Hence, the

⁶I focus on domestic households. Foreign agents maximize an identical utility function. They consume the same basket of goods as home agents, with identical parameters, and they are subject to similar constraints. I will sometimes refer to the representative consumer of generation v simply as the “representative consumer” below. It is understood that consumers of different generations can behave differently.

law of one price holds for each individual good and $p_t(i) = \varepsilon_t p_t^*(i)$, where ε_t is the exchange rate (units of domestic currency per unit of foreign) and $p_t^*(i)$ is the foreign currency price of good i . This hypothesis and identical intratemporal consumer preferences across countries ensure that consumption-based purchasing power parity (PPP) holds, *i.e.*, $P_t = \varepsilon_t P_t^*$.

The representative consumer enters a period holding bonds, money balances, and shares purchased in the previous period. She or he receives interests and dividends on these assets, may earn capital gains or incur losses on shares, earns labor income, is taxed, and consumes.

Denote the date t price (in units of domestic currency) of a claim to the representative domestic firm i 's entire future profits (starting on date $t+1$) by V_t^i . Let x_{t+1}^{vj} be the share of the representative domestic firm i owned by the representative domestic consumer j born in period v at the end of period t . D_t^i denotes the nominal dividends firm i issues on date t . Then, letting A_{t+1}^{vj} (A_{t+1}^{*vj}) be the home consumer's holdings of domestic (foreign) currency denominated bonds entering time $t+1$, the period budget constraint expressed in units of domestic currency is:

$$\begin{aligned} & P_t C_t^{vj} + P_t T_t^v + A_{t+1}^{vj} + \varepsilon_t A_{t+1}^{*vj} + \int_0^a V_t^i x_{t+1}^{vj} di + M_t^{vj} \\ &= (1 + i_t) A_t^{vj} + \varepsilon_t (1 + i_t^*) A_t^{*vj} + \int_0^a (V_t^i + D_t^i) x_t^{vj} di + M_{t-1}^{vj} + W_t L_t^{vj}, \end{aligned} \quad (2)$$

where i_t (i_t^*) is the nominal interest rate on holdings of domestic (foreign) bonds between $t-1$ and t , W_t is the nominal wage, M_{t-1}^{vj} denotes the agent's holdings of nominal money balances entering period t , and T_t^v is a lump-sum net real transfer, which is identical across members of generation v . Given that individuals are born owning no financial wealth, because not linked by altruism to individuals born in previous periods, $A_v^{vj} = A_v^{*vj} = x_v^{vj} = M_{v-1}^{vj} = 0$.

The representative domestic consumer born in period v maximizes the intertemporal utility function (1) subject to the constraint (2). Dropping the j superscript (because symmetric agents make identical choices in equilibrium), optimal labor supply is given by:

$$L_t^v = 1 - \frac{1 - \rho}{\rho} \frac{C_t^v}{w_t}, \quad (3)$$

which equates the marginal cost of supplying labor with the marginal utility of consumption generated by the corresponding increase in labor income (w_t denotes the real wage, $\frac{W_t}{P_t}$).

Making use of this equation, the first-order condition for optimal holdings of domestic currency bonds yields the Euler equation:

$$C_t^v = \left[\beta (1 + i_{t+1}) \left(\frac{P_t}{P_{t+1}} \right) \right]^{-1} C_{t+1}^v \quad (4)$$

for all $v \leq t$.

Demand for home currency real balances is:

$$\frac{M_t^v}{P_t} = \frac{\chi}{\rho} \frac{1 + i_{t+1}}{i_{t+1}} C_t^v. \quad (5)$$

Real domestic currency balances increase with consumption and decrease with the opportunity cost of holding money.

Condition (4) can be combined with the first-order condition for holdings of foreign bonds to yield a no-arbitrage condition between domestic and foreign currency bonds for domestic agents. Absence of unexploited arbitrage opportunities requires:

$$1 + i_{t+1} = (1 + i_{t+1}^*) \frac{\varepsilon_{t+1}}{\varepsilon_t}. \quad (6)$$

The consumption-based real interest rate between t and $t + 1$ is defined by the familiar Fisher parity condition:

$$1 + r_{t+1} = (1 + i_{t+1}) \frac{P_t}{P_{t+1}} = \frac{1 + i_{t+1}}{1 + \pi_{t+1}^{CPI}}, \quad (7)$$

where π_{t+1}^{CPI} is CPI inflation ($\pi_{t+1}^{CPI} \equiv \frac{P_{t+1}}{P_t} - 1$). PPP ensures that $1 + \pi_t^{CPI} = (1 + e_t) (1 + \pi_t^{CPI*})$, where $1 + \pi_t^{CPI*} \equiv \frac{P_t^*}{P_{t-1}^*}$ and $1 + e_t \equiv \frac{\varepsilon_t}{\varepsilon_{t-1}}$. Combining (7) with (6) and making use of PPP shows that $1 + r_{t+1} = 1 + r_{t+1}^* = (1 + i_{t+1}^*) \frac{P_t^*}{P_{t+1}^*}$: real interest rates are equal across countries in the absence of unexpected shocks that may cause no-arbitrage conditions to fail *ex post*.

Absence of arbitrage opportunities between bonds and shares in the domestic economy requires $1 + i_{t+1} = \frac{D_{t+1}^i + V_{t+1}^i}{V_t^i}$. Letting $d_t^i \equiv \frac{D_t^i}{P_t}$ and $v_t^i = \frac{V_t^i}{P_t}$, we can re-write this no-arbitrage condition as:

$$1 + r_{t+1} = \frac{d_{t+1}^i + v_{t+1}^i}{v_t^i}. \quad (8)$$

As usual, first-order conditions and the period budget constraint must be combined with appropriate transversality conditions to ensure optimality.

2.2 Firms

Output supplied at time t by the representative domestic firm i is a linear function of labor demanded by the firm:

$$Y_t^{Si} = Z_t L_t^i. \quad (9)$$

Z_t is an exogenous economy-wide productivity parameter. Production by the representative foreign firm is a linear function of L_t^{i*} , with productivity parameter Z_t^* .⁷

⁷Because all firms in the world economy are born at $t = -\infty$, after which no new goods appear, it is not necessary to index output and factor demands by the firms' date of birth. As for consumers, I focus on domestic firms below. Foreign firms are symmetric in all respects.

Output demand comes from several sources: domestic and foreign consumers and domestic and foreign firms. The demand for home good i by the representative home consumer born in period v is $c_t^v(i) = \left(\frac{p_t(i)}{P_{Ht}}\right)^{-\theta} \left(\frac{P_{Ht}}{P_t}\right)^{-\omega} C_t^v$, obtained by maximizing C^v subject to a spending constraint. *Total demand* for home good i coming from domestic consumers is:

$$\begin{aligned} c_t(i) &= a \left[\dots \frac{n}{(1+n)^{t+1}} c_t^{-t}(i) + \dots + \frac{n}{(1+n)^2} c_t^{-1}(i) + \frac{n}{1+n} c_t^0(i) \right] \\ &\quad + n c_t^1(i) + n(1+n) c_t^2(i) + \dots + n(1+n)^{t-1} c_t^t(i) \\ &= \left(\frac{p_t(i)}{P_{Ht}}\right)^{-\theta} \left(\frac{P_{Ht}}{P_t}\right)^{-\omega} a(1+n)^t c_t, \end{aligned} \quad (10)$$

where

$$c_t \equiv \frac{a \left[\dots \frac{n}{(1+n)^{t+1}} C_t^{-t} + \dots + \frac{n}{(1+n)^2} C_t^{-1} + \frac{n}{1+n} C_t^0 \right] + n C_t^1 + n(1+n) C_t^2 + \dots + n(1+n)^{t-1} C_t^t}{a(1+n)^t} \quad (11)$$

is aggregate per capita home consumption.

Given identity of intratemporal preferences, *total* demand for the same good by foreign consumers is $c_t^*(i) = \left(\frac{p_t(i)}{P_{Ht}}\right)^{-\theta} \left(\frac{P_{Ht}}{P_t}\right)^{-\omega} (1-a)(1+n)^t c_t^*$, where c_t^* is aggregate per capita foreign consumption.

Changing the price of its output is costly for the firm, which generates nominal rigidity. Specifically, I assume that the real cost (measured in units of the composite good) of output-price inflation volatility around a steady-state level of inflation equal to 0, is $PAC_t^i = \frac{\kappa}{2} \left(\frac{p_t(i)}{p_{t-1}(i)} - 1\right)^2 \frac{p_t(i)}{P_t} Y_t^i$. When the firm changes the price of its output, material goods—*e.g.*, new catalogs, price tags, etc.—need to be purchased. The price adjustment cost (PAC^i) captures the amount of marketing materials that must be purchased to implement a price change. Because the amount of these materials is likely to increase with firm size, PAC^i increases with revenues ($(p_t(i)/P_t)Y_t^i$), which are taken as a proxy for size. The cost is convex in inflation; faster price movements are more costly to the firm. I assume $\kappa \geq 0$. When $\kappa = 0$, prices are flexible. As Roberts (1995) pointed out, Rotemberg's (1982) quadratic cost of price adjustment yields dynamics for the aggregate economy that are similar to those resulting from staggered price setting as in Calvo (1983) and Yun (1996).

Total demand for good i produced in the home country is obtained by adding the demands for that good originating in the two countries. Making use of the results above, it is:

$$Y_t^{Di} = \left(\frac{p_t(i)}{P_{Ht}}\right)^{-\theta} \left(\frac{P_{Ht}}{P_t}\right)^{-\omega} Y_t^{DW}. \quad (12)$$

Y_t^{DW} is aggregate world demand of the composite good, defined as $Y_t^{DW} \equiv C_t^W + PAC_t^W$. $C_t^W \equiv (1+n)^t [a c_t + (1-a) c_t^*]$ and $PAC_t^W \equiv a PAC_t^i + (1-a) PAC_t^{*i}$ denote aggregate world consumption

and the world aggregate cost of adjusting prices, respectively.⁸

At time t_0 , firm i maximizes the present discounted value of dividends to be paid from t_0 on: $v_{t_0}^i + d_{t_0}^i = \sum_{s=t_0}^{\infty} R_{t_0,s} d_s^i$, where $R_{t_0,s} \equiv \frac{1}{\prod_{u=t_0+1}^s (1+r_u)}$, $R_{t_0,t_0} = 1$. Firm revenues are taxed at a constant, proportional rate τ . In addition, firms receive a lump-sum transfer (or tax) from the government, T_t^{fi} . At each point in time, dividends are given by real revenues, net of taxes, plus the lump-sum transfer, minus costs: $d_t^i = (1 - \tau) \frac{p_t(i)}{P_t} Y_t^i + T_t^{fi} - \left[\frac{W_t}{P_t} L_t^i + \frac{\kappa}{2} \left(\frac{p_t(i)}{p_{t-1}(i)} - 1 \right)^2 \frac{p_t(i)}{P_t} Y_t^i \right]$. The firm chooses the price of its product and the amount of labor demanded in order to maximize the present discounted value of its current and future profits subject to the constraints (9) and (12), and the market clearing condition $Y_t^i = Y_t^{Si} = Y_t^{Di}$. Firm i takes the aggregate price indexes, the wage rate, Z_t , world aggregates, and taxes and transfers as given.

Let λ_t^i denote the Lagrange multiplier on the constraint $Y_t^i = Y_t^{Di}$. Then, λ_t^i is the shadow price of an extra unit of output to be sold in period t , or the marginal cost of time t sales. The first-order condition with respect to $p_t(i)$ yields the pricing equation:

$$p_t(i) = \Psi_t^i P_t \lambda_t^i, \quad (13)$$

which equates the price charged by firm i to the product of the (nominal) shadow value of one extra unit of output—the (nominal) marginal cost ($P_t \lambda_t^i$)—and a markup (Ψ_t^i). The latter depends on output demand as well as on the impact of today’s pricing decision on today’s and tomorrow’s costs of adjusting the output price:

$$\Psi_t^i \equiv \theta Y_t^i \left\{ (\theta - 1) Y_t^i \left[1 - \tau - \frac{\kappa}{2} \left(\frac{p_t(i)}{p_{t-1}(i)} - 1 \right)^2 \right] + \kappa \Upsilon_t^i \right\}^{-1}, \quad (14)$$

where $\Upsilon_t^i \equiv Y_t^i \frac{p_t(i)}{p_{t-1}(i)} \left(\frac{p_t(i)}{p_{t-1}(i)} - 1 \right) - \frac{Y_{t+1}^i}{1+r_{t+1}} \frac{P_t}{P_{t+1}} \left(\frac{p_{t+1}(i)}{p_t(i)} \right)^2 \left(\frac{p_{t+1}(i)}{p_t(i)} - 1 \right)$ reflects the firm’s incentive to smooth price changes over time.

If $\kappa = 0$, *i.e.*, if prices are fully flexible, $\Psi_t^i = \theta / [(\theta - 1)(1 - \tau)]$, the familiar constant-elasticity markup. If $\kappa \neq 0$, price rigidity generates endogenous fluctuations of the markup. Firms react to CPI dynamics in their pricing decisions. Changes in monetary policy generate changes in CPI inflation. Hence, they affect producer prices and the markup. Through this channel, they generate different dynamics of relative prices and the real economy.

The first-order condition for the optimal choice of L_t^i yields:

$$\frac{W_t}{P_t} = \lambda_t^i Z_t. \quad (15)$$

⁸The expression for the world aggregate cost of adjusting prices derives from the assumption that the number of firms is constant. In the expression for PAC_t^W , I have already made use of the fact that symmetric firms make identical equilibrium choices. Keeping the i superscript for individual firms’ variables allows me to denote several aggregate per capita variables referring to firms by dropping the superscript below.

Today's real wage must equal the shadow value of an extra unit of output.

Using the market clearing conditions $Y_t^{Si} = Y_t^{Di}$ and $Y_t^{DW} = Y_t^{SW} = Y_t^W$, the expressions for supply and demand of good i , and recalling that symmetric firms make identical equilibrium choices (so that $p_t(i) = P_{Ht}$ and $p_t(i)$ is the producer price index, PPI) yields:

$$L_t^i = \left(\frac{p_t(i)}{P_t} \right)^{-\omega} \frac{Y_t^W}{Z_t}. \quad (16)$$

Firm i 's labor demand is a decreasing function of the relative price of good i and of labor productivity. It is an increasing function of world demand of the composite good. Henceforth, I denote the relative price of good i by $RP_t^i \equiv \frac{p_t(i)}{P_t}$.

2.3 The government

I assume that governments in both countries run balanced budgets. The government taxes firm revenues at a rate that compensates for monopoly power in a zero-inflation steady state and removes the markup over marginal cost charged by firms in a flexible-price world. The tax rate is determined by $1 - \tau = \frac{\theta}{\theta - 1}$, which yields $\tau = -\frac{1}{\theta - 1}$. Because the tax rate is negative, firms receive a subsidy on their revenues and pay lump-sum taxes determined by $T_t^{fi} = \tau RP_t^i Y_t^i$. In addition, the government injects money into the economy through lump-sum transfers of seignorage revenues to households: $P_t T_t^v = -\left(M_t^{vj} - M_{t-1}^{vj} \right)$. Similarly for the foreign government.

2.4 Aggregation and equilibrium

2.4.1 Households

Aggregate per capita consumption and labor supply are obtained by aggregating consumption and labor supply across generations and dividing by total population at each point in time. Aggregate per capita labor supplies follow from aggregating the labor-leisure tradeoffs in the two economies:

$$L_t = 1 - \frac{1 - \rho}{\rho} \frac{c_t}{w_t}, \quad L_t^* = 1 - \frac{1 - \rho}{\rho} \frac{c_t^*}{w_t^*}. \quad (17)$$

Consumption Euler equations in aggregate per capita terms contain an adjustment for consumption by the newborn generation at time $t + 1$:

$$c_t = \frac{1 + n}{\beta(1 + r_{t+1})} \left(c_{t+1} - \frac{n}{1 + n} C_{t+1}^{t+1} \right), \quad c_t^* = \frac{1 + n}{\beta(1 + r_{t+1})} \left(c_{t+1}^* - \frac{n}{1 + n} C_{t+1}^{t+1*} \right). \quad (18)$$

Newborn households hold no assets, but they own the present discounted value of their labor income. I define human wealth, h_t , as the present discounted value of the household's lifetime

endowment of time in terms of the real wage: $h_t \equiv \sum_{s=t}^{+\infty} R_{t,s} w_s$, $h_t^* \equiv \sum_{s=t}^{+\infty} R_{t,s} w_s^*$. The dynamics of h and h^* are described by the following forward-looking difference equations:

$$h_t = \frac{h_{t+1}}{1 + r_{t+1}} + w_t, \quad h_t^* = \frac{h_{t+1}^*}{1 + r_{t+1}} + w_t^*. \quad (19)$$

Using the labor-leisure tradeoff (3), the Euler equation (4), and a newborn household's intertemporal budget constraint, it is possible to show that the household's consumption in the first period of its life is a fraction of the household's human wealth at birth:

$$C_{t+1}^{t+1} = \rho(1 - \beta) h_{t+1}, \quad C_{t+1}^{t+1*} = \rho(1 - \beta) h_{t+1}^*. \quad (20)$$

Aggregate per capita real money demands in the two economies are:

$$m_t \equiv \frac{M_t}{P_t} = \frac{\chi}{\rho} \frac{1 + i_{t+1}}{i_{t+1}} c_t, \quad m_t^* \equiv \frac{M_t^*}{P_t^*} = \frac{\chi}{\rho} \frac{1 + i_{t+1}^*}{i_{t+1}^*} c_t^*. \quad (21)$$

2.4.2 Firms

Aggregate per capita, real GDP in each economy is obtained by expressing production of each differentiated good in units of the composite basket, multiplying by the number of firms, and dividing by population. (Translation of units of each differentiated good into units of the consumption basket is necessary for a sensible aggregation of outputs of differentiated goods.) In equilibrium, $RP_t^i = RP_t$ and similarly for foreign firms. Thus:

$$y_t = RP_t Z_t L_t, \quad y_t^* = RP_t^* Z_t^* L_t^*. \quad (22)$$

For given employment and productivity, real GDP rises with the relative price of the representative good produced, as this is worth more units of the consumption basket. (In a closed economy model, symmetry of the equilibrium implies $RP_t = 1$, so that the relative price does not appear in the expression for aggregate, real GDP.)

Aggregate per capita labor demand is:

$$L_t = RP_t^{-\omega} \frac{y_t^W}{Z_t}, \quad L_t^* = RP_t^{*-\omega} \frac{y_t^W}{Z_t^*}, \quad (23)$$

where y_t^W is aggregate per capita world production of the composite good, equal to aggregate per capita world consumption plus the aggregate per capita resource cost of price changes, $c_t^W + pac_t^W$. It is $y_t^W = ay_t + (1 - a)y_t^*$, $c_t^W = ac_t + (1 - a)c_t^*$, $pac_t^W = apac_t + (1 - a)pac_t^*$. Market clearing requires $y_t^W = c_t^W + pac_t^W$.

Domestic and foreign relative prices are equal to markups over marginal costs:

$$RP_t = \Psi_t \frac{w_t}{Z_t}, \quad RP_t^* = \Psi_t^* \frac{w_t^*}{Z_t^*} \quad (24)$$

2.4.3 Equity and bonds

In the absence of arbitrage opportunities between bonds and shares, the aggregate per capita equity values of the home and foreign economies entering period $t + 1$ must evolve according to:

$$v_t = \frac{1+n}{1+r_{t+1}}v_{t+1} + \frac{d_{t+1}}{1+r_{t+1}}, \quad v_t^* = \frac{1+n}{1+r_{t+1}}v_{t+1}^* + \frac{d_{t+1}^*}{1+r_{t+1}}. \quad (25)$$

where $v_t \equiv \frac{aV_t^i}{P_t N_{t+1}}$, $v_t^* \equiv \frac{aV_t^{*i}}{P_t^* N_{t+1}^*}$, and d_t and d_t^* denote aggregate per capita real dividends, equal to $(1-\tau)y_t + T_t^f - w_t L_t - pac_t$ and $(1-\tau^*)y_t^* + T_t^{f*} - w_t^* L_t^* - pac_t^*$, respectively (note that $\tau = \tau^*$). y_t (y_t^*) denotes domestic (foreign) aggregate per capita, real GDP, defined below. pac_t (pac_t^*) is the aggregate per capita cost of nominal rigidity at home (abroad).

The law of motion of aggregate per capita net foreign assets is obtained by aggregating an equilibrium version of the budget constraint (2) across generations alive at each point in time. It is:

$$\begin{aligned} (1+n)B_{t+1} &= (1+r_t)B_t + w_t L_t + d_t - c_t, \\ (1+n)B_{t+1}^* &= (1+r_t)B_t^* + w_t^* L_t^* + d_t^* - c_t^*. \end{aligned} \quad (26)$$

where $B_{t+1} \equiv \frac{A_{t+1} + \varepsilon_t A_{t+1}^*}{P_t}$ and $B_{t+1}^* \equiv \frac{A_{t+1}^* + A_{t+1}^*}{P_t^*}$ denote domestic and foreign net bond holdings (A_* denotes foreign households' holdings of home bonds, A_*^* denotes their holdings of foreign bonds). A country's net foreign assets and net foreign bond holdings coincide in a world in which all shares are held domestically.⁹

Because $d_t = y_t - w_t L_t - pac_t$ and $d_t^* = y_t^* - w_t^* L_t^* - pac_t^*$ in equilibrium, the equations in (26) become:

$$\begin{aligned} (1+n)B_{t+1} &= (1+r_t)B_t + y_t - c_t - pac_t, \\ (1+n)B_{t+1}^* &= (1+r_t)B_t^* + y_t^* - c_t^* - pac_t^*. \end{aligned} \quad (27)$$

2.4.4 International equilibrium

For international asset markets to be in equilibrium, net, aggregate home assets (liabilities) must equal net, aggregate foreign liabilities (assets). In terms of aggregates per capita, it must be $aB_t + (1-a)B_t^* = 0$. Using this condition, the equations in (27) reduce to $y_t^W = c_t^W + pac_t^W$:

⁹Strictly speaking, these equations hold in all periods *after* the initial one. The UIP condition may be violated between time $t_0 - 1$ and t_0 if an unexpected shock surprises agents at the beginning of period t_0 . Using log-linear versions of these equations to determine asset accumulation in the initial period is harmless if one is willing to assume that the steady-state levels of A , A^* , A_* , and A_*^* are all zero. (The model pins down the steady-state levels of B and B^* endogenously. Because domestic and foreign bonds are perfect substitutes once no-arbitrage conditions are met, the model does not pin down the levels of A , A^* , A_* , and A_*^* .)

consistent with Walras' Law, asset market equilibrium implies goods market equilibrium, and vice versa.

2.5 The steady state

The procedure for finding the steady-state levels of real variables follows the same steps as in Ghironi (2000) and Cavallo and Ghironi (2001). I refer to those papers for details. As described there, the departure from Ricardian equivalence caused by entry of new households with no assets in each period generates dependence of aggregate per capita consumption growth on the stock of aggregate per capita net foreign assets. This yields determinacy of steady-state real net foreign asset holdings, and thus of the steady-state levels of other real variables in the model.

I denote steady-state levels of variables with overbars and assume that the economy is in steady state up to and including time -1 below. Unexpected shocks can surprise agents at the beginning of period 0, generating the dynamics described in the following sections.

Given steady-state levels of productivity ($\bar{Z} = \bar{Z}^* = 1$) and inflation ($\bar{\pi}^{PPI} = \bar{\pi}^{PPI^*} = \bar{\pi}^{CPI} = \bar{\pi}^{CPI^*} = 0$, where $\pi_t^{PPI} \equiv \frac{p_t(i) - p_{t-1}(i)}{p_{t-1}(i)}$ and $\pi_t^{PPI^*}$ is defined similarly), real variables are stationary, in the sense that they return to the initial position determined below following non-permanent shocks to productivity and/or inflation. Steady-state levels of real variables are:

$$\begin{aligned}\bar{w} &= \bar{RP} = \bar{w}^* = \bar{RP}^* = 1, & \bar{h} &= \bar{h}^* = \frac{1}{1-\beta}, \\ \bar{y} &= \bar{c} = \bar{C}_v^u = \bar{L} = \bar{y}^* = \bar{c}^* = \bar{C}_v^{u*} = \bar{L}^* = \bar{y}^W = \bar{c}^W = \rho, \\ \bar{\Psi} &= \bar{\Psi}^* = 1, & \bar{pac} &= \bar{d} = \bar{v} = \bar{pac}^* = \bar{d}^* = \bar{v}^* = 0.\end{aligned}$$

Steady-state nominal interest rates and real money balances are $\bar{i} = \bar{i}^* = \frac{1-\beta}{\beta}$ and $\bar{m} = \bar{m}^* = \frac{\chi}{1-\beta}$, respectively. Given initial, steady-state levels of money supply $\bar{M} = \bar{M}^* = \frac{\chi}{1-\beta}$, it is $\bar{\varepsilon} = \bar{P} = \bar{P}^* = \bar{p}(h) = \bar{p}^*(f) = 1$ ($\bar{p}(h)$ and $\bar{p}^*(f)$ are the domestic and foreign PPIs, respectively, and their steady state levels follow from $\bar{RP} = \frac{\bar{p}(h)}{\bar{P}} = \bar{RP}^* = \frac{\bar{p}(f)}{\bar{P}^*} = 1$). The model does not pin down the steady-state levels of all nominal variables endogenously as function of the structural parameters only. An exogenous choice of steady-state money supply levels is necessary. As a consequence, monetary policy may generate the presence of a unit root in the dynamics of price levels, the exchange rate, and nominal money balances. Steady-state levels of nominal variables may change as a consequence of temporary shocks depending on the nature of monetary policy.

3 The log-linear system

The equations that determine domestic and foreign variables can be log-linearized around the steady state. I use sans serif fonts to denote percentage deviations from the steady state. Percentage

deviations of inflation, depreciation, and interest rates from the steady state refer to gross rates. From now on, π denotes the percentage deviation of the corresponding (gross) inflation rate from the steady state. It is convenient to solve the model for cross-country differences ($x_t^D \equiv x_t - x_t^*$ for any variable x) and world averages ($x_t^W \equiv ax_t + (1-a)x_t^*$). The levels of individual country variables can be recovered given solutions for differences and world averages.

3.1 No-arbitrage conditions

PPP implies that the CPI inflation differential equals exchange rate depreciation:

$$\pi_t^{CPI^D} = e_t, \quad (28)$$

where $e_t \equiv \epsilon_t - \epsilon_{t-1}$ and ϵ denotes the percentage deviation of ε from the steady state.

Uncovered interest parity (UIP) implies:

$$i_{t+1}^D = \epsilon_{t+1} - \epsilon_t. \quad (29)$$

3.2 Households

The relative labor-leisure tradeoff is:

$$w_t^D = c_t^D + \frac{\rho}{1-\rho} L_t^D. \quad (30)$$

Log-linear Euler equations and consumption functions for newborn households imply that the consumption differential obeys:

$$c_t^D = (1+n)c_{t+1}^D - nh_{t+1}^D, \quad (31)$$

where h is the deviation of human wealth from the steady state. The *ex ante* real interest rate has no effect, because agents in both countries face identical real rates. The random walk result of the standard Obstfeld-Rogoff (1995) model for real variables is transparent here. If $n = 0$, *i.e.*, if no new agents with zero assets enter the economy, the consumption differential between the two countries follows a random walk. Any shock that causes a consumption differential today has permanent consequences on the relative level of consumption. When $n > 0$, the Euler equation is adjusted for consumption of a newborn generation in the first period of its life ($C_t^D = h_t^D$). The human wealth differential, h^D , is determined by:

$$h_t^D = \beta h_{t+1}^D + (1-\beta)w_t^D. \quad (32)$$

3.3 Firms

The GDP differential obeys:

$$y_t^D = \text{RP}_t^D + \text{L}_t^D + \text{Z}_t^D. \quad (33)$$

I assume $\text{Z}_t = \phi \text{Z}_{t-1}$, $\text{Z}_t^* = \phi \text{Z}_{t-1}^*$, $\forall t > 0$, $0 \leq \phi < 1$. Hence, $\text{Z}_t^D = \phi \text{Z}_{t-1}^D$.

The relative price differential reflects relative markup and marginal cost dynamics:

$$\text{RP}_t^D = \psi_t^D + \text{w}_t^D - \text{Z}_t^D, \quad (34)$$

where ψ denotes the percentage deviation of the markup (Ψ) from the steady state. Defining the domestic terms of trade following Obstfeld and Rogoff (1995) as $\frac{p(h)}{\varepsilon p^*(f)}$, it is easy to verify that RP^D is the percentage deviation of the terms of trade from the steady state.

The difference between domestic and foreign labor demand depends on the markup differential and on relative marginal cost and productivity:

$$\text{L}_t^D = -\omega (\psi_t^D + \text{w}_t^D - \text{Z}_t^D) - \text{Z}_t^D. \quad (35)$$

Substituting equations (34) and (35) into (33) yields an expression for the GDP differential as a function of relative markup and cost dynamics:

$$y_t^D = -(\omega - 1) (\psi_t^D + \text{w}_t^D - \text{Z}_t^D). \quad (36)$$

Combining labor demand (35) with the labor-leisure tradeoff (30) yields the equilibrium real wage differential:

$$\text{w}_t^D = \frac{1}{1 + \rho(\omega - 1)} [(1 - \rho) c_t^D - \rho \omega \psi_t^D + \rho(\omega - 1) \text{Z}_t^D]. \quad (37)$$

From firms' optimal pricing (equation (13) for domestic firms and the analogous equation for foreign), the PPI inflation differential depends positively on the CPI inflation differential and on relative markup and marginal cost growth:

$$\pi_t^{PPI^D} = \pi_t^{CPI^D} + \psi_t^D - \psi_{t-1}^D + \text{w}_t^D - \text{w}_{t-1}^D - (\text{Z}_t^D - \text{Z}_{t-1}^D). \quad (38)$$

Alternatively, the PPI inflation differential can be written as a function of nominal depreciation and relative real GDP growth, if $\omega \neq 1$:

$$\pi_t^{PPI^D} = \epsilon_t - \epsilon_{t-1} - \frac{1}{\omega - 1} (y_t^D - y_{t-1}^D) \quad (39)$$

Finally, using $1 - \tau = 1 - \tau^* = \frac{\theta}{\theta - 1}$ and the definitions of domestic and foreign markups, relative markup dynamics depend on current and future pricing decisions:

$$\psi_t^D = -\frac{\kappa}{\theta} \left[\pi_t^{PPI^D} - \beta(1 + n) \pi_{t+1}^{PPI^D} \right]. \quad (40)$$

3.4 Asset accumulation

Log-linearizing the laws of motion for real net foreign assets of domestic and foreign households, subtracting the resulting equation for foreign assets from that for home assets, and imposing the log-linear bond market equilibrium condition, $a\mathbf{B}_t + (1 - a)\mathbf{B}_t^* = 0$, yields:

$$\mathbf{B}_{t+1} = \frac{1}{1+n} \left[\frac{1}{\beta} \mathbf{B}_t + (1-a)(y_t^D - c_t^D) \right]. \quad (41)$$

Accumulation of aggregate per capita domestic net foreign assets is faster (slower) the larger (smaller) the GDP (consumption) differential. (Because $\bar{B}_0 = \bar{B}_0^* = 0$, \mathbf{B} and \mathbf{B}^* are defined as percentage deviations of B and B^* from the steady-state level of domestic and foreign consumption, respectively.)

The dynamics of the relative equity value (relative stock market dynamics) reflect the relative behavior of the markup in the two economies (see Cavallo and Ghironi, 2001, for details):

$$v_t^D = \beta(1+n)v_{t+1}^D + \beta\psi_{t+1}^D. \quad (42)$$

3.5 World averages

The world labor-leisure tradeoff is:

$$w_t^W = c_t^W + \frac{\rho}{1-\rho} l_t^W. \quad (43)$$

Averaging log-linear Euler equations across countries and using the (log-linear) consumption functions for newborn households and the equilibrium condition $c_t^W = y_t^W$ yields:

$$y_t^W = -r_{t+1} + (1+n)y_{t+1}^W - nh_{t+1}^W, \quad (44)$$

where r is the percentage deviation of the gross real interest rate from the steady state, which satisfies:

$$r_{t+1} = i_{t+1}^W - \pi_{t+1}^{CPI^W}. \quad (45)$$

Equation (44) resembles the standard, forward-looking IS relation of the recent closed economy literature, the only difference being inclusion of world human wealth, which disappears if $n = 0$. World human wealth obeys:

$$h_t^W = -\beta r_{t+1} + \beta h_{t+1}^W + (1-\beta)w_t^W. \quad (46)$$

It is easy to verify that $RP^W = 0$, because $P^W = aP + (1-a)P^* = ap(h) + (1-a)p^*(f) = p^W(i)$, *i.e.*, there is no difference between world CPI and world PPI. Thus, the world production function is:

$$y_t^W = Z_t^W + L_t^W. \quad (47)$$

with $Z_t^W = \phi Z_{t-1}^W$. World marginal cost equals the negative of the markup:

$$w_t^W - Z_t^W = -\psi_t^W. \quad (48)$$

And the dynamics of the markup are described by:

$$\psi_t^W = -\frac{\kappa}{\theta} \left[\pi_t^{CPI^W} - \beta(1+n)\pi_{t+1}^{CPI^W} \right], \quad (49)$$

because $\pi_t^{PPI^W} = \pi_t^{CPI^W}$. Equations (48) and (49) relate inflation to expected inflation and current marginal cost as in the Calvo-Yun setup.

3.6 Monetary policy

I assume that central banks set interest rates according to simple Taylor-type rules of the form:

$$i_{t+1} = \alpha_1 y_t + \alpha_2 \pi_t^{CPI} + \xi_t, \quad i_{t+1}^* = \alpha_1 y_t^* + \alpha_2 \pi_t^{CPI^*} + \xi_t^*, \quad (50)$$

with $\alpha_1 \geq 0$, $\alpha_2 > 1$. (Recall that i_{t+1} and i_{t+1}^* are set at time t .) The reaction coefficients to GDP and inflation are identical at home and abroad. Because the two economies are identical in all structural features, if central banks with identical objectives independently chose the optimal values of α_1 and α_2 , they would choose identical reaction coefficients. ξ and ξ^* are exogenous interest rate shocks. I assume $\xi_t = \mu \xi_{t-1}$, $\xi_t^* = \mu \xi_{t-1}^*$, $\forall t > 0$ ($t = 0$ being the time of an initial, surprise impulse below), $0 \leq \mu < 1$. Hence, $\xi_t^D = \mu \xi_{t-1}^D$.

The interest rate rules in (50) yield $i_{t+1}^D = \alpha_1 y_t^D + \alpha_2 \pi_t^{CPI^D} + \xi_t^D$. Because PPP implies $\pi_t^{CPI^D} = \epsilon_t - \epsilon_{t-1}$, it is:

$$i_{t+1}^D = \alpha_1 y_t^D + \alpha_2 (\epsilon_t - \epsilon_{t-1}) + \xi_t^D. \quad (51)$$

The world monetary stance is described by:

$$i_{t+1}^W = \alpha_1 y_t^W + \alpha_2 \pi_t^{CPI^W} + \xi_t^W, \quad (52)$$

with $\xi_t^W = \mu \xi_{t-1}^W$.

Before moving on, I stress that nominal interest rates react to the deviations of GDP from the steady state rather than to the output gap—the deviation of GDP from the flexible price equilibrium—in the benchmark policy specification of this paper. This is consistent with Taylor's (1993) original

analysis. But a reaction to the output gap is the standard in the recent normative literature on monetary policy. I stick to the Taylor benchmark for essentially two reasons. First, this is a positive, rather than normative, paper. Its purpose is to point out how reasonable assumptions about openness, parameter values, and asset markets can help explain dynamics that are observed in U.S. data that the Taylor-specification fits fairly well. (Ireland, 2001, considers a similar specification, though he allows for a reaction of the interest rate to money growth. Clarida and Gertler, 1997, and Clarida, Galí, and Gertler, 1998, provide evidence from other countries.) Second, the normative claim that central banks should react to the output gap is borne out of representative agent models subject to rather stringent assumptions. It is not clear that the same result would hold here.¹⁰

4 World dynamics

The domestic economy coincides with the world economy if $a = 1$. In this case, the home economy is a closed economy. Regardless of the relative size of home and foreign, the world economy *is* a closed economy by definition. This implies that we can use the system (43)-(49), (52) to describe closed economy dynamics. Straightforward substitutions yield the following system of equations for world inflation, GDP, and human wealth:

$$\pi_t^{CPI^W} = \beta(1+n)\pi_{t+1}^{CPI^W} + \frac{\theta}{\kappa(1-\rho)}(y_t^W - Z_t^W), \quad (53)$$

$$y_t^W = \frac{1}{1+\alpha_1} \left[(1+n)y_{t+1}^W - nh_{t+1}^W + \pi_{t+1}^{CPI^W} - \alpha_2\pi_t^{CPI^W} - \xi_t^W \right], \quad (54)$$

$$h_t^W = \beta h_{t+1}^W + \left(\frac{1-\beta}{1-\rho} - \beta\alpha_1 \right) y_t^W + \beta \left(\pi_{t+1}^{CPI^W} - \alpha_2\pi_t^{CPI^W} \right) - \frac{\rho(1-\beta)}{1-\rho} Z_t^W - \beta\xi_t^W. \quad (55)$$

Equation (53) is familiar, forward-looking, New Keynesian Phillips curve for world inflation, the only difference relative to the standard equation being the adjustment of discounting for population growth. Equation (54) follows from substituting the world policy rule (52) into the IS equation (44). Similarly, the dynamic equation for human wealth incorporates the assumptions about monetary policy.¹¹

¹⁰I do not allow for interest rate smoothing in the policy rule because I do not want to introduce persistence in the model economy through this channel. In addition to the assumptions about interest rate setting, I assume that speculative bubbles in prices or the exchange rate are ruled out by the commitment to fractional backing mechanisms as in Obstfeld and Rogoff (1983).

¹¹If there is no population growth, the behavior of world inflation and GDP under policy (52) is described by the very familiar equations:

$$\begin{aligned} \pi_t^{CPI^W} &= \beta\pi_{t+1}^{CPI^W} + \frac{\theta}{\kappa(1-\rho)}(y_t^W - Z_t^W), \\ y_t^W &= \frac{1}{1+\alpha_1} \left(y_{t+1}^W + \pi_{t+1}^{CPI^W} - \alpha_2\pi_t^{CPI^W} - \xi_t^W \right). \end{aligned}$$

The system (53)-(55) can be combined with $Z_t^W = \phi Z_{t-1}^W$ and $\xi_t^W = \mu \xi_{t-1}^W$ to solve for the dynamics of the closed, world economy. I assume that the reaction of central banks to inflation is such that the system has a determinate equilibrium. The solution can be written as:

$$\pi_t^{CPI^W} = \eta_{\pi^W Z^W} Z_t^W + \eta_{\pi^W \xi^W} \xi_t^W, \quad (56)$$

$$y_t^W = \eta_{y^W Z^W} Z_t^W + \eta_{y^W \xi^W} \xi_t^W, \quad (57)$$

$$h_t^W = \eta_{h^W Z^W} Z_t^W + \eta_{h^W \xi^W} \xi_t^W, \quad (58)$$

where the η 's are elasticities that can be obtained with the method of undetermined coefficients as in Campbell (1994).

Equations (56)-(58) imply that there is no persistence in world inflation, GDP, and human wealth beyond that implied by the persistence of the productivity and policy shocks. In particular, equation (57) shows that the standard Calvo-Yun-Rotemberg sticky-price model cannot generate endogenous persistence of closed economy GDP dynamics, as well as of other real variables. This is the weakness of the closed economy benchmark highlighted in Clarida, Galí, and Gertler (1999) and Chari, Kehoe, and McGrattan (2000). In addition, monotone dynamics of Z_t^W and ξ_t^W imply that the closed economy benchmark cannot generate hump-shaped responses to shocks in line with the data regardless of the persistence of the shocks. Real GDP dynamics are monotonic, lacking the hump-shaped behavior that motivates Christiano, Eichenbaum, and Evans (2001) to introduce habit persistence and Papadopoulou (2001) to allow for limited participation.

5 Cross-country differences

5.1 Flexible prices

To understand better the impact of price stickiness under internationally incomplete markets, I start by briefly discussing the solution of the model under flexible prices. If prices are flexible ($\kappa = 0$), a dichotomy exists between nominal and real variables in the model. Real variables affect nominal ones, but the converse is not true (except for real balances, which are a function of the nominal interest rate). There is no longer a time-varying, forward-looking markup. Equilibrium profits are zero in all periods, along with the equity value of both economies. The equations that describe firm behavior in the log-linear system for cross-country differences between real variables simplify as $\psi_t^D = d_t^D = v_t^D = 0 \forall t$.

Given the simplified, flexible-price system, it is easy to show that $c_t^D = w_t^D = L_t^D = y_t^D = 0$ if $\omega = 1$. Unitary intratemporal elasticity of substitution ensures that domestic and foreign consumption, the real wage, employment, and GDP are equal regardless of productivity. Hence, to preserve bond market equilibrium, it must be $B_t = B_t^* = 0$ if $\omega = 1$. This is the result first obtained

by Corsetti and Pesenti (2001). If the elasticity of substitution between domestic and foreign goods is one, accumulation of net foreign assets plays no role in the transmission of shocks, and current accounts are always zero: $y_t = c_t$ and $y_t^* = c_t^*$. This is the same allocation that would be generated by the complete markets assumption, through perfect “risk-sharing” between the domestic and foreign economy.

To solve the model, observe that aggregating the consumption functions for individual domestic and foreign households and log-linearizing yields the following expression for the consumption differential:

$$c_t^D = \frac{\rho(1-\beta)}{\beta(1-a)} B_t + h_t^D. \quad (59)$$

The consumption differential in each period reflects the net foreign asset position of the two economies and the differential between the expected real wage paths from that period on.

Using (59) in conjunction with the flexible-price versions of (32), (36), (37), and (41) yields:

$$B_{t+1} = \gamma_1 B_t - \gamma_2 h_t^D + \gamma_3 Z_t^D, \quad (60)$$

$$h_t^D = \gamma_4 h_{t+1}^D + \gamma_5 B_t + \gamma_6 Z_t^D, \quad (61)$$

where

$$\begin{aligned} \gamma_1 &\equiv \frac{1 + \rho(\omega\beta - 1)}{\beta(1+n)[1 + \rho(\omega - 1)]}, & \gamma_2 &\equiv \frac{\omega(1-a)}{(1+n)[1 + \rho(\omega - 1)]}, & \gamma_3 &\equiv \frac{(\omega - 1)(1-a)}{(1+n)[1 + \rho(\omega - 1)]}, \\ \gamma_4 &\equiv \frac{\beta[1 + \rho(\omega - 1)]}{\rho\omega + \beta(1-\rho)}, & \gamma_5 &\equiv \frac{\rho(1-\rho)(1-\beta)^2}{\beta(1-a)[\rho\omega + \beta(1-\rho)]}, & \gamma_6 &\equiv \frac{\rho(1-\beta)(\omega - 1)}{\rho\omega + \beta(1-\rho)}. \end{aligned}$$

Equations (60) and (61) constitute a system of two equations in two unknowns (the endogenous state variable B and the forward-looking variable h^D) plus the exogenous relative productivity term Z^D . The stock of net foreign assets and the levels of the exogenous productivity parameters describe the state of the (real) economy in each period. I assume that the restrictions on structural parameter values such that the solution of the system (60)-(61) exists and is unique are satisfied. The solution can be written as:

$$B_{t+1} = \eta_{BB} B_t + \eta_{BZ^D} Z_t^D, \quad (62)$$

$$h_t^D = \eta_{h^D B} B_t + \eta_{h^D Z^D} Z_t^D. \quad (63)$$

In addition, the solution for the GDP differential can be written as:

$$y_t^D = \eta_{y^D B} B_t + \eta_{y^D Z^D} Z_t^D. \quad (64)$$

The cross-country GDP differential displays persistence beyond that of the exogenous productivity differential because of persistent net foreign asset dynamics. Since η_{BB} is close to 1 for realistic

parameter values, relative productivity shocks cause a persistent GDP differential far beyond the time at which the exogenous shock has died out *under flexible prices*. (See Ghironi, 2000, and Cavallo and Ghironi, 2001.) No such persistence would arise under internationally complete asset markets or unitary intratemporal elasticity of substitution between domestic and foreign goods. If $\omega = 1$, $\eta_{y^D B} = \eta_{y^D Z^D} = \eta_{B Z^D} = 0$ (and $\mathbf{B}_t = \mathbf{y}_t^D = 0 \forall t$).

Given that domestic GDP is given by $y_t = y_t^W + (1 - a)y_t^D$, a relative productivity shock has endogenously persistent consequences on domestic GDP as long as $a < 1$ and $\omega \neq 1$. Nevertheless, the elasticity $\eta_{y^D B}$ is quite small for realistic values of structural parameters. This implies that persistent net foreign asset dynamics under flexible prices are not sufficient to generate *sizeable* persistent deviations of domestic GDP from the steady state following exogenous shocks. Once the shock has died out, the deviation of domestic GDP from the steady state becomes very small.¹²

5.2 Sticky prices

The dynamics of the economy are now affected by the markup fluctuations generated by nominal rigidity.

It is possible to prove that $\omega = 1$ implies $\mathbf{B}_{t+1} = 0 \forall t$ also under sticky prices regardless of other parameter values. Intuitively, equation (36) shows that the GDP differential is always zero regardless of productivity and interest rates if the elasticity of substitution between domestic and foreign goods is one. Because countries are starting off with zero net assets, identical GDP levels imply that the two economies have identical real resources to allocate to consumption in all periods. Thus, the utility maximizing choice entails $\mathbf{c}_t^D = 0 \forall t$.

The path of the nominal exchange rate can be determined by using the UIP condition (29) in conjunction with the interest setting rules for the domestic and foreign economy. Combining equation (51) with UIP and rearranging, we obtain:

$$\epsilon_{t+1} - (1 + \alpha_2)\epsilon_t + \alpha_2\epsilon_{t-1} = \alpha_1 y_t^D + \xi_t^D. \quad (65)$$

The system on which I focus for the general case $\omega \neq 1$ consists of equations (31), (32), (36), (37), (39), (40), (41), (42), and (65), combined with the assumptions on the shock processes, $\xi_t^D = \mu \xi_{t-1}^D$ and $\mathbf{Z}_t^D = \phi \mathbf{Z}_{t-1}^D$. It is hard to obtain an easily interpretable analytical solution for this system. Thus, I resort to Uhlig's (1999) numerical implementation of Campbell (1994). The endogenous state vector is $[\mathbf{B}_{t+1}, \epsilon_t, \psi_t^D, \mathbf{w}_t^D, \mathbf{y}_t^D, \mathbf{h}_t^D, \mathbf{v}_t^D]'$, the vector of other endogenous variables is $[\pi_t^{PPID}, \mathbf{c}_t^D]'$, and the vector of exogenous driving forces is $[\mathbf{Z}_t^D, \xi_t^D]'$. I include \mathbf{h}_t^D and \mathbf{v}_t^D in the state vector to avoid singularity problems in the solution. I interpret periods as quarters and

¹²Inclusion of capital accumulation in the model along the lines of Backus, Kehoe, and Kydland (1994) would likely amplify the importance of net foreign asset accumulation for cross-country differentials under flexible prices.

consider the following benchmark parameter values: $\beta = .99$ (a standard value), $\rho = .33$ (in steady state, agents spend one-third of their time working), $a = .5$ (the two economies—say, the U.S. and Europe—have equal size), $\omega = 3$, and $n = .01$ (population grows by one percent per quarter). A value of 3 for the elasticity of substitution between domestic and foreign goods is in line with estimates from the trade literature, indeed a conservative choice. (For example, see Harrigan, 1993, Shiells, Stern, and Deardorff, 1986, and Trefler and Lai, 1999) The choice of n is higher than realistic, at least if one has developed economies in mind and n is interpreted strictly as the rate of growth of population.¹³ However, I could reproduce the same speed of return to the steady state with slower population growth in a version of the model that incorporates probability of not surviving as in Blanchard (1985). I take $n = .01$ as a proxy for that situation. I assume $\alpha_1 = .5$ and $\alpha_2 = 1.5$, as in the interest rule popularized by Taylor (1993). Finally, I set κ to 77, the estimate in Ireland (2001) for the post-1979 period, and θ to 6, consistent with Rotemberg and Woodford (1992). These values imply that PPI inflation of 1 percent would generate a resource cost of .385 percent of aggregate per capita real GDP. The choice of θ implies a steady-state markup (non-adjusted for the subsidy τ) of 20 percent.

Table 1 shows the solution for the relevant elasticities for different values of the persistence parameters ϕ and μ .¹⁴ Nominal price rigidity implies that all variables in the endogenous state vector (with the exception of the exchange rate) can be written as functions of \mathbf{B}_t , y_{t-1}^D , and the shocks only (*i.e.*, that the values of all other elasticities are zero, at least for the benchmark parameterization and a number of plausible alternatives).¹⁵ In particular, the solution for the GDP differential can now be written as:

$$y_t^D = \eta_{y^D y^D} y_{t-1}^D + \eta_{y^D B} \mathbf{B}_t + \eta_{y^D Z^D} \mathbf{Z}_t^D + \eta_{y^D \xi^D} \xi_t^D. \quad (66)$$

Sticky prices introduce persistence in the relative GDP process (and in other real variables) beyond its dependence on assets accumulated in the previous period. Recall that GDP is measured in units of the consumption basket, *i.e.*, by multiplying production of domestic goods by their relative price. Stickiness in the latter generates endogenous persistence in the GDP differential and, hence, in the dynamics of each country's GDP. The assumption that markets are internationally incomplete (and that the elasticity of substitution between domestic and foreign goods differs from 1) is crucial for this result. If $\omega = 1$, there is no GDP differential at any point in time. Hence, $y_t = y_t^* = y_t^W \forall t$ and there is no persistence in GDP dynamics. Similarly, openness is crucial for endogenous persistence. If home coincides with the world economy ($a = 1$), $y_t = y_t^W \forall t$ regardless of market incompleteness.

¹³The average rate of quarterly population growth for the U.S. between 1973:1 and 2000:3 has been .0025.

¹⁴The table includes also key elasticities for world dynamics.

¹⁵In addition, the exchange rate features a unit root.

6 Quantitative analysis

Table 1 shows that the elasticity of today's GDP differential to its past value, $\eta_{y^D y^D}$, is sizable for very reasonable parameter values. The endogenous persistence effect of price stickiness and market incompleteness on GDP dynamics is significant.

To evaluate the quantitative significance of the persistence generated by price stickiness and market incompleteness, Figure 1 presents impulse responses for three measures of real activity following a 1 percent impulse to domestic productivity with persistence $\phi = 0, .5, .9$. I consider impulse responses for y^W (this would be the response of output to a .5 percent productivity shock in the closed economy model), y (domestic GDP in units of consumption basket), and $y(h)$ (domestic output in units of the representative domestic good). I include the latter as it shows that persistent relative price dynamics translate into endogenous persistence of output also when the *direct* effect of relative price stickiness is removed.¹⁶

The top portion of Figure 1 shows that world GDP is above the steady state for only one period following a productivity shock with no persistence. Market incompleteness under sticky prices amplifies the initial deviation and causes domestic output to be above the steady state for several periods.¹⁷ World GDP deviates more persistently from the steady state if $\phi = .5$. However, the deviations of y and $y(h)$ remain larger and more persistent. The bottom portion of Figure 1 shows that sticky prices and incomplete markets succeed in generating hump-shaped responses of y and $y(h)$ for sufficiently high persistence of the productivity shock. Instead, the response of closed-economy GDP is monotonic regardless of the persistence of productivity.

Figure 2 repeats the exercise for the case of a 1 percent impulse to the domestic interest rate. The pattern is similar to that for productivity shocks. Sticky prices and incomplete markets magnify output responses and generate hump-shaped dynamics for sufficiently persistent shocks. There can be no hump in the closed economy case. In both figures 1 and 2, a value of ω above 1 but quite low in the range provided by the trade literature generates substantial differences in GDP dynamics relative to the $\omega = 1$ /internationally complete markets allocation. This reinforces Ghironi's (2000) conclusion that incomplete markets, open economy dynamics can be quite different from those under complete markets, different from results in Heathcote and Perri (2002), Kehoe and Perri (2002), and Schmitt-Grohé and Uribe (2001).

The quantitative relevance of stickiness, openness, and market incompleteness hinges on the

¹⁶This happens because persistence in a country's real consumption resources affects the dynamics of the labor effort in general equilibrium.

¹⁷Indeed, the deviations of y and $y(h)$ from the steady state are more persistent than it is apparent from Figure 1. Small, non-zero deviations persist until net foreign assets remain different from zero. This is the case for a large number of periods, as a small rate of entry of new households implies slow convergence of net foreign assets to the steady state.

assumptions about κ (the size of the cost of price adjustment), a (the relative size of the economy), and ω (the elasticity of substitution between domestic and foreign goods). The smaller κ , the larger a , and the closer ω to 1 (the closer the allocation to that under internationally complete markets), the smaller the endogenous persistence effect of stickiness, openness, and market incompleteness. However, the results of this section suggest that price stickiness succeeds at generating sizable endogenous persistence in real dynamics when combined with openness of the economy and market incompleteness for quite reasonable parameter values.

7 Conclusions

I have used a two-country, monetary model to show that endogenously persistent dynamics of the real economy are not a puzzle for the Calvo-Yun-Rotemberg sticky-price benchmark if the economy is open and asset markets are incomplete. Market incompleteness matters because complete markets would remove endogenous persistence through persistent dynamics of cross-country differentials generated by sticky relative prices. Openness is crucial because, if the home economy coincides with the world economy, domestic GDP coincides with world GDP regardless of asset market structure.¹⁸ The endogenous persistence effect of incomplete markets in a standard sticky-price model is quantitatively sizable for reasonable parameter values. The model can explain hump-shaped dynamics of the real economy following productivity and monetary policy shocks. Extending the model to incorporate capital accumulation with reasonable costs of adjusting the capital stock in each country would reinforce the conclusions.

Openness and market incompleteness provide both an alternative and a complement to Christiano, Eichenbaum, and Evans' (2001) habit formation, costly capital accumulation, and variable capital utilization and to Papadopoulou's (2001) limited participation as ingredients of sticky-price models that can explain important features of the data.

¹⁸Stationary net foreign asset dynamics are necessary to avoid spurious persistence of the open economy under incomplete markets.

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Table 1. The sticky-price solution, elasticities
Endogenous states to endogenous states

	B	ϵ	ψ^D	w^D	y^D	h^D	v^D
B	.9940	0	0	0	.2144	0	0
ϵ	.0050	1	0	0	-.2041	0	0
ψ^D	-.0042	0	0	0	-.5438	0	0
w^D	.0060	0	0	0	.3256	0	0
y^D	-.0036	0	0	0	.4363	0	0
h^D	.0023	0	0	0	.0066	0	0
v^D	-.0830	0	0	0	-.4498	0	0

Endogenous states to shocks

	$\phi = 0$	$\mu = 0$	$\phi = .5$	$\mu = .5$	$\phi = .9$	$\mu = .9$
	Z^D	ξ^D	Z^D	ξ^D	Z^D	ξ^D
B	.0499	-.2858	.0727	-.3128	.1513	-.2206
ϵ	-.0475	-.3946	-.1047	-.5500	-.3879	-1.1087
ψ^D	1.3661	.7250	1.3054	.8041	1.0556	.6223
w^D	-.4168	-.4342	-.3800	-.4834	-.2214	-.3838
y^D	.01015	-.5817	.1492	-.6414	.3317	-.4770
h^D	-.0034	-.0088	-.0053	-.0192	.0026	-.0683
v^D	-.1046	.5997	.9849	2.1177	5.9911	10.4412

Other endogenous variables to endogenous states

	B	ϵ	ψ^D	w^D	y^D	h^D	v^D
π^{PPI^D}	.0069	0	0	0	.0778	0	0
c^D	.0086	0	0	0	.0033	0	0

Other endogenous variables to shocks

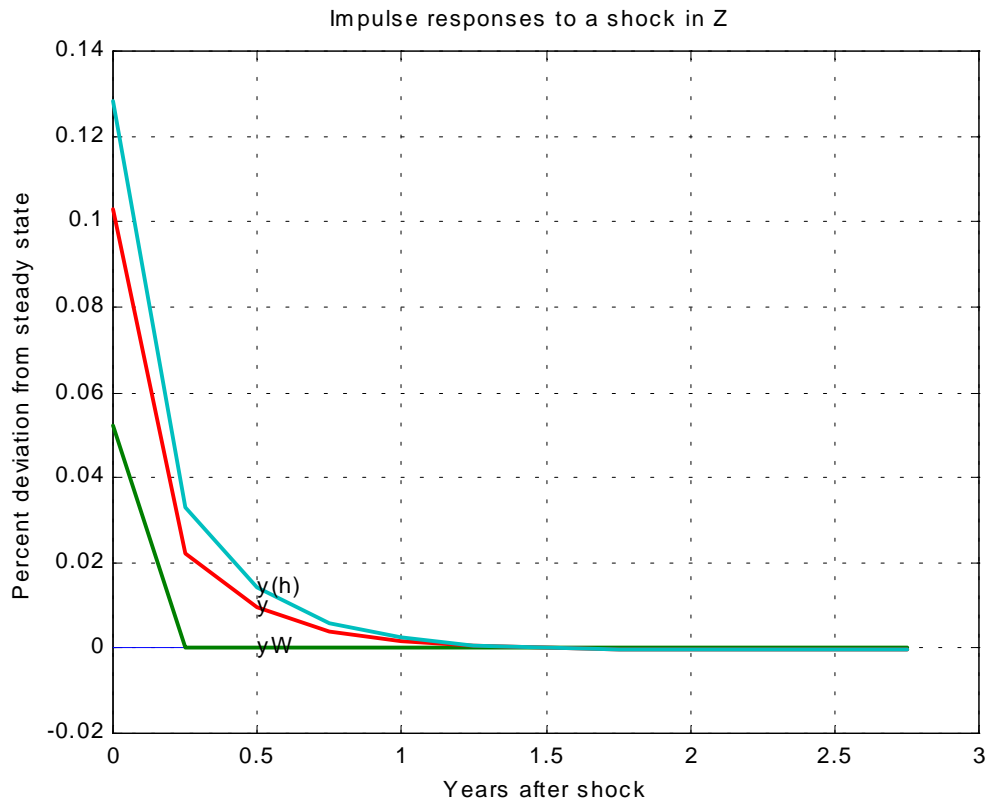
	$\phi = 0$	$\mu = 0$	$\phi = .5$	$\mu = .5$	$\phi = .9$	$\mu = .9$
	Z^D	ξ^D	Z^D	ξ^D	Z^D	ξ^D
π^{PPI^D}	-.0982	-.1037	-.1792	-.2293	-.5538	-.8702
c^D	.0008	-.0044	.0023	-.0095	.0261	-.0314

World GDP and inflation to shocks

	$\phi = 0$	$\mu = 0$	$\phi = .5$	$\mu = .5$	$\phi = .9$	$\mu = .9$
	Z^W	ξ^W	Z^W	ξ^W	Z^W	ξ^W
y^W	.1042	-.5972	.1887	-.8113	.5376	-.7706
π^{CPI^W}	-.1042	-.0695	-.1887	-.1887	-.5373	-.8955

Figure 1. Impulse responses, domestic productivity shock

$\phi = 0$



$\phi = .5$

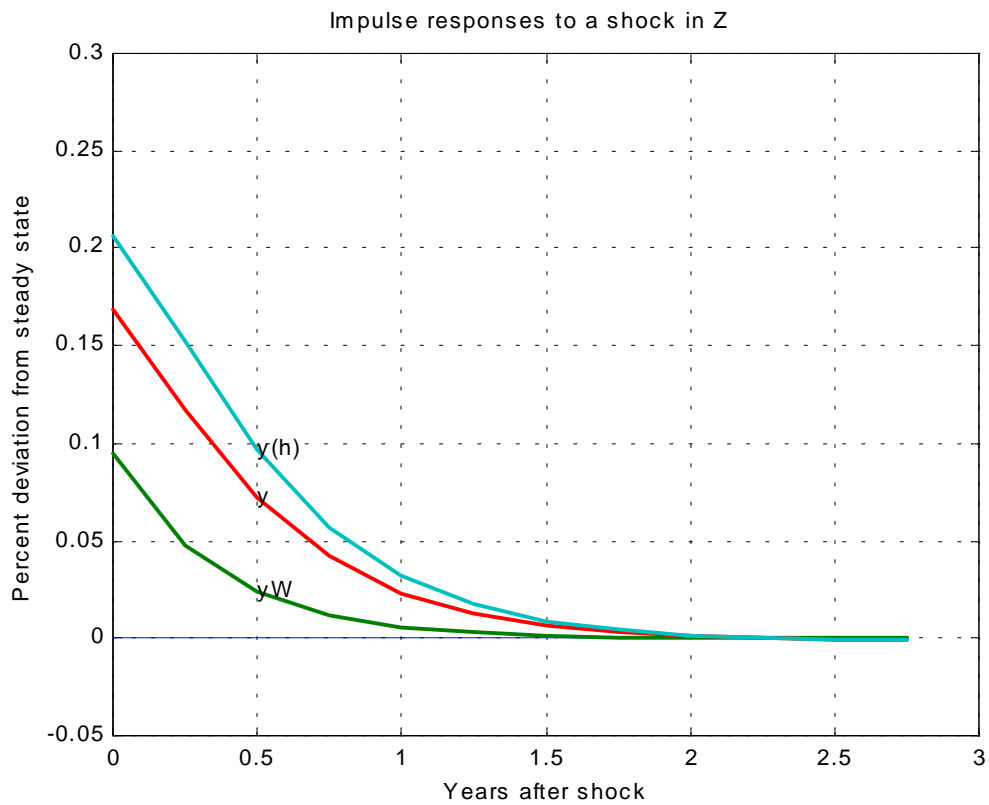


Figure 1, continued

$\phi = .9$

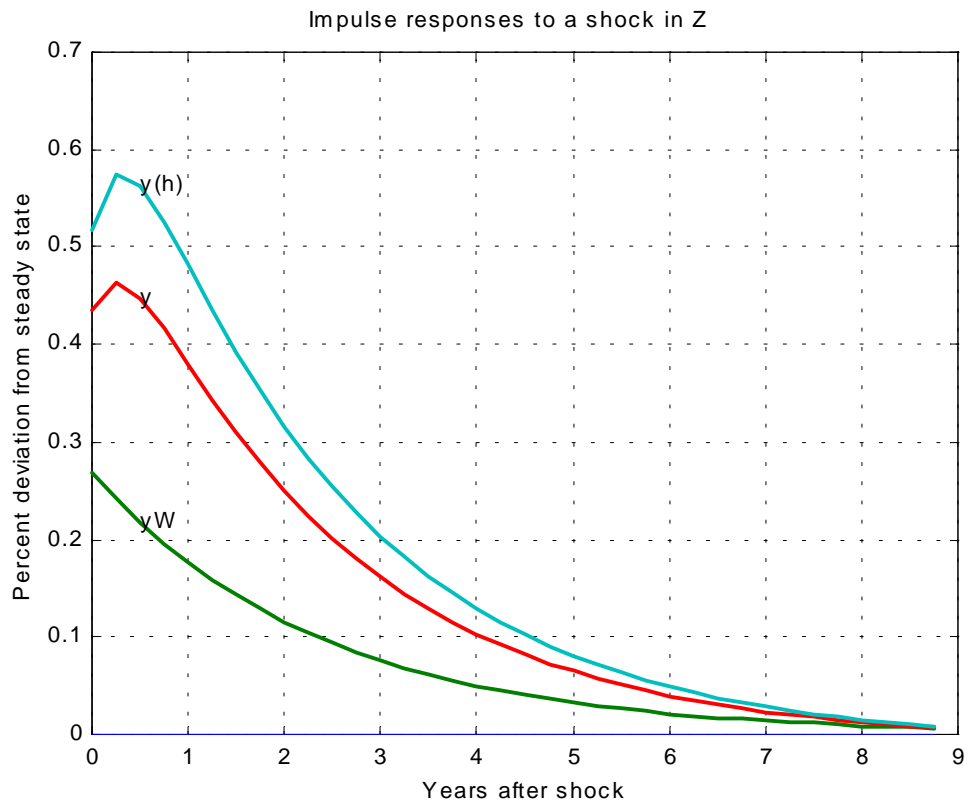
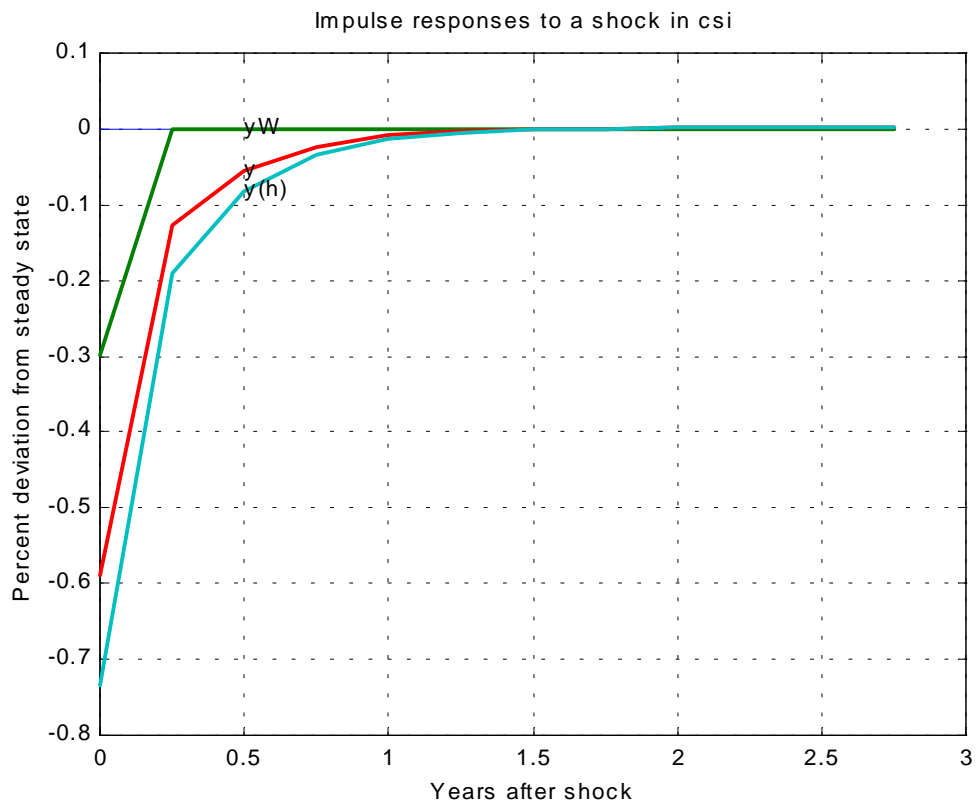


Figure 2. Impulse responses, domestic monetary policy shock

$\mu = 0$



$\mu = .5$

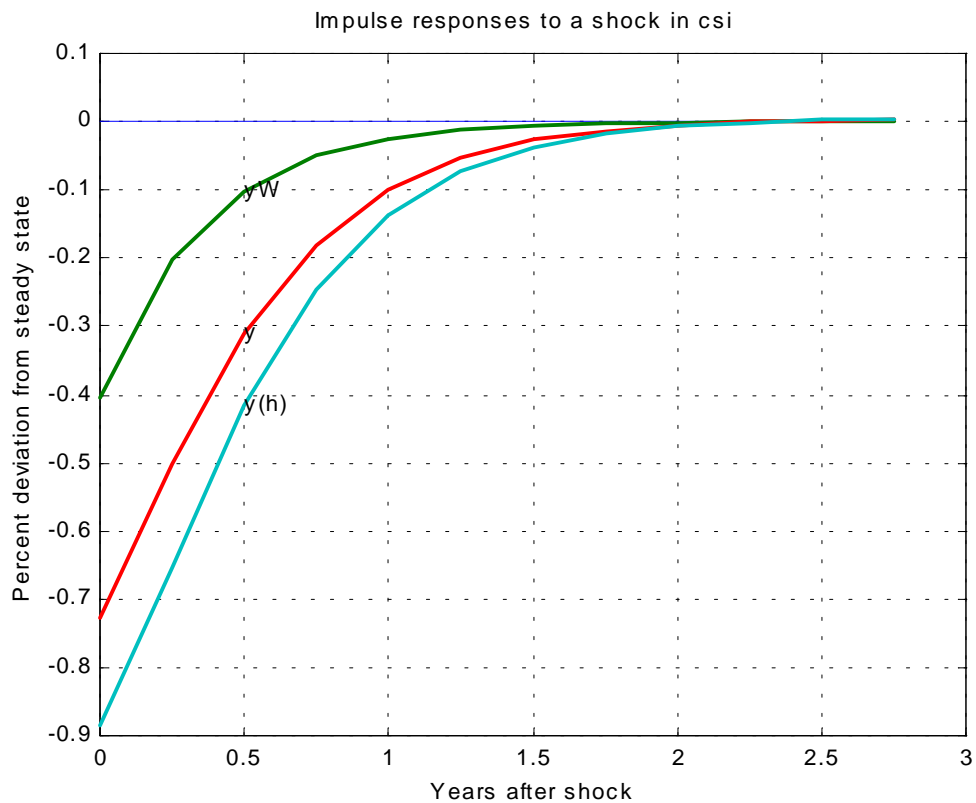


Figure 2, continued

$\mu = .9$

