

*Economics Department*  
*Working Papers in Economics*

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*Boston College*

*Year 2000*

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Towards New Open Economy  
Macroeconometrics

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# Towards New Open Economy Macroeconometrics \*

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First draft: August 9, 1999  
This draft: February 23, 2000

## Abstract

I estimate the structural parameters of a small open economy model using data from Canada and the United States. The model improves upon the recent literature in open economy macroeconomics from an empirical perspective. I estimate parameters by using non-linear least squares at the single-equation level. Estimates of most parameters are characterized by small standard errors and are in line with the findings of other studies. I also develop a plausible way of constructing measures for non-observable variables. To verify if multiple-equation regressions yield significantly different estimates, I run full information maximum likelihood, system-wide regressions. The results of the two procedures are similar. Finally, I illustrate a practical application of the model, showing how a shock to the U.S. economy is transmitted to Canada under an inflation targeting monetary regime.

*Keywords:* Open economy macroeconometrics; Stationarity; Shocks transmission

*JEL Classification:* C51, F41

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\* This paper is based on Chapter 3 of my Ph.D. Dissertation at U.C. Berkeley. It contains part of the material I presented in my job seminars in 1999. I thank the members of my Dissertation Committee—Barry Eichengreen, Maury Obstfeld, David Romer, and Andy Rose—as well as David Card and Francesco Giavazzi, for their suggestions and encouragement. I thank for comments and useful conversations Gianluca Benigno, Laura Bottazzi, Petra Geraats, and Paolo Manasse. I am also grateful to seminar audiences at Berkeley, the Board of Governors, Boston College, Brown, Florida International University, George Washington University, Iowa State, Johns Hopkins, the New York Fed, NYU, Tilburg University, UCLA, University of Pittsburgh, and Washington University. I thank George di Giovanni, Julian di Giovanni, and Philip Oreopoulos for helping me overcome the vagaries of the Internet to gather data on the Canadian economy in the Summer of 1998. Andrei Levchenko provided outstanding research assistance. Remaining errors are of course my own. Financial support from the Institute of International Studies at U.C. Berkeley and the MacArthur Foundation during academic years 1997/1998 and 1998/1999 is gratefully acknowledged. Much work on this paper was undertaken while I was at the Federal Reserve Bank of New York.

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## 1. Introduction

I estimate the structural parameters of a small open economy model using data from Canada and the United States. The model improves upon the recent literature in open economy macroeconomics from an empirical perspective. It is described in detail in Ghironi (1999a). I estimate parameters by using non-linear least squares at the single-equation level. Calibration is used only when the regressions do not yield sensible estimates. The sensibility of the parameter values obtained in this way is verified by comparing them to the findings of a large empirical literature. Estimates of most parameters turn out to be characterized by small standard errors and are in line with the findings of other studies. Illustrating a plausible way of constructing measures for non-observable variables is a contribution of this paper. To verify if multiple-equation regressions yield significantly different estimates, I also run full information maximum likelihood system-wide regressions taking the estimates from the single-equation procedure as initial values. The results of the two procedures are similar.

I then illustrate the functioning of the model by using the parameter estimates to calibrate it and analyze the transmission of a shock to U.S. GDP to the Canadian economy under inflation targeting, the monetary regime currently followed by the Bank of Canada. When doing this exercise, I combine the theoretical model of the Canadian economy with a simple VAR that traces the comovements of U.S. variables affecting Canada directly. The exercise illustrates the role of markup and relative price dynamics in the model. The latter does a better job than the flexible-price framework used by Schmitt-Grohé (1998) at explaining the transmission of U.S. cycles to Canada.<sup>1</sup>

The structure of the paper is as follows. Section 2 outlines the main features of the model and compares it to the existing literature. Section 3 presents the main log-linear equations of the model and illustrates the estimation procedure. Section 4 is devoted to the example. Section 5 concludes.

## 2. The Model and the Literature

After a long-lasting predominance of non-microfounded Keynesian models, the publication of Obstfeld and Rogoff's "Exchange Rate Dynamics Redux" in 1995 opened the way to a new generation of models of macroeconomic interdependence. These models combine a rigorously microfounded approach with analytical tractability. The literature following Obstfeld and Rogoff's work has been mainly theoretical. Papers in this literature are said to belong to the so-called "new open economy macroeconomics."<sup>2</sup> However, empirical performance will ultimately decide whether this new generation of models will supplant the time-honored Mundell-Fleming-Dornbusch framework as the main tool for understanding interdependence and for formulating policy advice. This paper is a contribution in that direction. It provides—to the best of my knowledge—the first comprehensive attempt at estimating an open-economy model in line with the recent developments in the theoretical literature and it illustrates how to use the model in an empirical way. For this reason, the paper can be thought of as an initial contribution to "new open economy macroeconometrics."

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<sup>1</sup> In Ghironi (1999b), I evaluate the performance of the Canadian economy under alternative monetary rules when Canada is subject to different sources of volatility. Because the exercise relies on estimates of the structural parameters of the model, the bearing of the Lucas critique on the results is weakened.

<sup>2</sup> For a survey, see Lane (1999).

After Obstfeld and Rogoff (1995) the theoretical literature in open economy macroeconomics has been evolving along different directions depending also on the role attributed to the current account. The latter plays a crucial role in the transmission of shocks in the original Obstfeld-Rogoff model. But this comes with a major problem of the framework that makes the conclusions questionable from a theoretical and empirical perspective, namely the absence of a well defined endogenously determined steady state. In the model, the position of the domestic and foreign economies that is taken to be the steady state in the absence of shocks is a point to which the economies never return following a disturbance. The consumption differential between countries follows a random walk. So do an economy's net foreign assets. Whatever level of asset holdings materializes in the period immediately following a shock becomes the new long-run position of the current account, until a new shock happens.

Determinacy of the steady state and stationarity fail because the average rate of growth of the economies' consumption in the model does not depend on average holdings of net foreign assets. Hence, setting consumption to be constant is not sufficient to pin down a steady-state distribution of asset holdings. This makes the choice of the economy's initial position for the purpose of analyzing the consequences of a shock arbitrary. When the model is log-linearized, one is actually approximating its dynamics around a "moving steady state." The reliability of the log-linear approximation is low, especially for analyses whose time horizon is longer than a one-period exercise, because variables wander away from the initial steady state. *De facto*, one cannot perform any stochastic analysis. The inherent unit root problem complicates empirical testing. And the long-run non-neutrality of money that characterizes the results can be attacked on empirical grounds.

Scholars of international macroeconomics soon recognized the problem. Some decided to dismiss it.<sup>3</sup> Others tried to finesse the current account issue in various ways. For example, Corsetti and Pesenti (1998) propose a model in which the importance of the current account is deemphasized. They achieve this by assuming unitary intratemporal elasticity of substitution between domestic and foreign goods in consumption. Under this assumption, the current account does not react to shocks, and thus plays no role in their transmission. The justification Corsetti and Pesenti offer for claiming that this is not a bad approximation of reality when the purpose is providing normative conclusions is that, even when the current account does move, the difference its movements make for a country's welfare is only second order. The Corsetti-Pesenti simplification has proved very successful in the literature. It makes it possible to solve the model without having to resort to log-linearization—a big tractability gain—and yields interesting theoretical insights. Several papers have later adopted the approach, including Benigno P. (1999) and Obstfeld and Rogoff (1998, 2000).

However, the Corsetti-Pesenti model shares the indeterminacy of the steady state with the original Obstfeld-Rogoff framework. There too, setting consumption to be constant does not pin down a steady-state international distribution of asset holdings, for the same reason mentioned above. The choice of a zero-asset initial equilibrium, combined with the assumption on the elasticity of substitution between foreign and domestic goods, allows Corsetti and Pesenti—and the followers of their approach—to shut off the current account channel. This opens the way to stochastic analysis in a highly tractable framework, but at a large cost in terms of realism. Any initial position different from the zero-asset equilibrium brings the non-stationarity of the model back to the surface.

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<sup>3</sup> I do so in Ghironi (2000).

An alternative way of dealing with the non-stationarity problem by shutting off the current account channel consists of assuming that financial markets are complete. Because of perfect risk sharing in complete markets, the current account does not react to shocks. This is the approach followed by Benigno G. (1999) and Gali and Monacelli (1999). This too yields highly tractable models suitable for stochastic analysis but at a potentially high cost in terms of realism. As argued by Obstfeld and Rogoff (1995, 1996 Ch. 10), the complete markets assumption also seems at odds with the presence of real effects of unanticipated monetary shocks in a world in which prices are sticky.

The Corsetti-Pesenti simplification or the complete markets assumption are now setting the trend in the theoretical literature. But there are reasons to believe that these approaches risk missing very important features of economic interdependence. For example, the recent dynamics of the U.S. current account suggest that the latter may play an important role in generating interdependence between the United States and the rest of the world. Hence, one may want to be able to analyze interdependence formally without shutting off the current account while at the same time preserving a reasonable degree of tractability.

In Ghironi (1999a), I develop a *tractable* perfect-foresight, two-country, general equilibrium model that offers a solution to the indeterminacy/non-stationarity issue that is not shutting off the current account channel.

I do so by changing the demographic structure relative to the representative agent framework used in most of the literature. I follow Weil (1989) in assuming that the world economy consists of distinct infinitely lived households that come into being on different dates and are born owning no assets. This demographic structure, combined with the assumption that newly born agents have no financial wealth, allows the model to be characterized by a steady state to which the world economy returns over time following temporary shocks. Agents consume; hold money balances, bonds, and shares in firms; and supply labor.

Because the model is stationary around an endogenously determined steady state, I do not rely on the Corsetti-Pesenti simplifying assumption that removes current account effects nor I assume complete markets. The current account does play a role in my model. Whether or not this is important can then be subject of empirical investigation.

As do most models in the recent open macro literature, I assume that a continuum of goods is produced in the world by monopolistically competitive firms, each of which produces a single differentiated good. Preferences for consumption goods are identical in the two countries, and the law of one price and consumption-based Purchasing Power Parity (PPP) hold.<sup>4</sup>

The failure of stationarity is not the only problem of several existing models. The assumption of one-period price rigidity that characterizes several exercises is not appealing for empirical purposes. The absence of investment and capital accumulation from the models limits their appropriateness for thorough empirical investigations of current-account behavior and of the consequences of alternative policy rules for medium to long-run dynamics.<sup>5</sup>

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<sup>4</sup> Engel and Rogers (1996) provide evidence of deviations from the law of one price between the U.S. and Canada—the economies on which I focus in my empirical exercise. This notwithstanding, I limit myself to the simpler case, to focus on other directions along which the original Obstfeld-Rogoff framework can be extended.

<sup>5</sup> Bergin (1997) extends the Obstfeld-Rogoff framework to allow for investment and capital accumulation and performs calibration exercises. Kollmann (1999) analyzes the implications of nominal rigidity for the behavior of asset prices. Nonetheless, the arbitrariness of the point around which to log-linearize is not resolved in their models.

In the model I propose, firms produce using labor and physical capital. Capital is accumulated via investment, and new capital is costly to install as in a familiar Tobin's  $q$  model. The presence of monopoly power has consequences for the dynamics of employment, by introducing a wedge between the real wage index and the marginal product of labor.

I assume that firms face costs of adjusting the price of their outputs. I choose a quadratic specification for these costs, as in Rotemberg (1982). This specification produces aggregate dynamics similar to those induced by staggered price setting a' la Calvo (1982). It also generates a markup endogenous to the conditions of the economy as long as the latter is not in steady state.<sup>6</sup> The dynamics of the markup play an important role in business cycle fluctuations, consistent with the analysis of Rotemberg and Woodford (1990). The dynamics of the real wage are not tied to those of the marginal product of labor.

My model lends itself naturally to empirical analysis and stochastic applications. Although the model is potentially a tool for analyzing bilateral interdependence between countries, in its first empirical implementation, I focus on a small open economy case in order to make use of a set of simplifying exogeneity assumptions. The home economy is identified with Canada, which is small and open when compared to the rest-of-the-world economy, approximated by the United States. For this reason, when presenting the model, I assume that the home economy is much smaller than the foreign one. The small open economy assumption implies that foreign variables and world aggregates are given from the perspective of the domestic economy. Exogeneity of foreign variables with respect to home's provides a set of restrictions I use in my empirical analysis.

### 3. Estimating the Log-Linear Economy

The microfounded setup of the model can be found in Ghironi (1999*a*), along with the solution for the steady state. In this section, I present the log-linear equations that govern the dynamics of the home economy—Canada—following small perturbations to the steady state and I illustrate a plausible approach to the estimation of the structural parameters of the model. For consistency with Ghironi (1999*b*), where I focus on monetary issues for Canada, I assume that fiscal policy variables are kept at their steady-state levels in what follows, and drop them from the log-linearized equations. Unobservable variables appear in some of these equations. An important part of this section deals with my empirical approach to the issue of measuring these variables.

The strategy used to estimate the structural parameters of the model consists of two steps. I first run single equation regressions. These—and the use of calibration when the regressions fail to yield sensible results—provide me with a set of baseline estimates. To verify the reliability of these estimates, I use them as starting values for full information maximum likelihood regressions of the systems of the firms' and consumers' first-order conditions.

Rotemberg and Woodford (1997) pursue an alternative approach to the issue of estimating a dynamic microfounded model. They run a three-variate recursive VAR for the U.S. economy and estimate the structural parameters of their model by calibrating them so that the model's impulse responses match the VAR's. I did not follow this strategy for two reasons. First, I find it ad hoc. Second, Rotemberg and Woodford's model is significantly simpler than mine, and it involves a smaller number of parameters. If I had followed their strategy, I would have faced the

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<sup>6</sup> Carré and Collard (1997) adopt a similar approach, as well as Ireland (1997, 1999) in a closed economy setting. G. Benigno (1999) and P. Benigno (1999) rely on Calvo-type mechanisms.

problem that a possibly large number of combinations of sensible parameter values are likely to allow the model to match the VAR's responses.

Cushman and Zha (1997) use the small open economy assumption to estimate a structural VAR of the U.S. and Canadian economies. Block exogeneity helps identify Canadian monetary shocks. However, no underlying model is estimated. Working along their lines, I could have run a large scale identified VAR a' la Leeper, Sims, and Zha (1996), but this would have left me with the problem of mapping the estimates of the VAR coefficients into estimates of the structural parameters of my model.

For these reasons, the strategy I decided to follow seems to be a reasonable way to let the data reveal something about parameter values. As it turns out, the strategy is fairly successful. As the reader will find out, I have to rely on pure calibration only in one case in the single-equation procedure, and the system-wide regressions yield estimates that are similar to those obtained in the first step. In addition, the estimates are in line with the findings of a large body of empirical literature.

### 3.a. Investment, Pricing, and Labor Demand

The log-linear production function is:

$$y_t = RP_t + \gamma k_t + (1 - \gamma)L_t + Z_t, \quad (3.1)$$

where arial variables denote percentage deviations from the initial steady state,  $y$  is detrended aggregate per capita GDP,  $RP$  is the price of the representative domestic good in terms of the consumption bundle ( $RP \equiv p(i)/P$ ),  $k$  is detrended aggregate per capita capital,  $L$  is aggregate per capita labor demand, and  $Z$  is a productivity shock.

The elasticity of Canadian output with respect to hours worked— $1 - \gamma$ —is the first structural parameter I estimate. The data come from the CANSIM database and from the IMF International Financial Statistics. I use quarterly data, consistent with the purpose of working at business cycle frequencies. Availability problems for some crucial series force me to restrict attention to the sample period 1980:1-1997:4. I construct the trend series  $E_t = (1 + g)E_{t-1}$  by assuming that  $g$  is the average rate of growth of Canadian real GDP per capita during the sample period and letting  $E_{1980:2} = 1$ . Steady-state levels of variables are calculated as the unconditional means of the series over the sample period. Variables in the regressions are defined as percentage deviations from the steady state, to match the concepts in the model.

Regressing Canadian GDP on the ratio of the industrial price index to the CPI, on the capital stock, on hours, and on a set of seasonal dummies, yields an estimate for  $1 - \gamma$  of approximately .9.<sup>7</sup> This is a high value for the elasticity of output to hours. However, it seems reasonable that GDP be much more sensitive to hours than capital on a quarterly basis. For this reason, I will take .9 to be the baseline value of  $1 - \gamma$  in what follows.

The log-linear equation that determines detrended investment at each point in time is:

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<sup>7</sup> The standard error of the estimate is .067, so that the estimated coefficient is highly significant. The estimated coefficient on capital in an unrestricted regression is .06, and it is hardly significant. The  $R^2$  of the regression is .82, and the Durbin-Watson statistic is 1.3. Alternative specifications of the regression improved the significance of capital and raised the Durbin-Watson statistic somewhat, yielding estimates for  $1 - \gamma$  in the range .86 to .93.

$$inv_t - k_t = \frac{1 + \eta[(1+n)(1+g) - (1-\delta)]}{\eta[(1+n)(1+g) - (1-\delta)]} q_t. \quad (3.2)$$

$n$  is the rate of growth of population,  $\eta$  measures the size of the cost of adjusting the capital stock,  $\delta$  is the rate of depreciation of capital, and  $q$  is Tobin's  $q$ . From the law of motion for detrended aggregate per capita capital, it follows that:

$$k_{t+1} - k_t = \frac{(1+n)(1+g) - (1-\delta)}{(1+n)(1+g)} (inv_t - k_t). \quad (3.3)$$

The change in aggregate per capita capital between  $t$  and  $t+1$  is faster the larger investment and the smaller the stock of capital at time  $t$ .

Log-linearizing the Euler equation for capital accumulation in detrended aggregate per capita terms gives:

$$q_t = -\tilde{r}_{t+1} + \frac{1-\delta}{1+r} q_{t+1} + \frac{\gamma}{1-\gamma} \frac{\bar{w}_0 \bar{L}_0}{\bar{q}_0 \bar{k}_0 (1+r)} (w_{t+1} + L_{t+1} - k_{t+1}) + \frac{\eta[(1+n)(1+g) - (1-\delta)]^2}{\bar{q}_0 (1+r)} (inv_{t+1} - k_{t+1}), \quad (3.4)$$

where  $\tilde{r}$  is the real interest rate—the steady-state level of which is  $r$ —and  $w$  is the detrended real wage.<sup>8</sup> A bar and the subscript 0 denote the constant steady-state level of the corresponding variable.

Tobin's  $q$  is one of the variables in the model that pose significant measurement problems. I construct a measure for the economy-wide  $q$  for Canada as follows.

Given the expression for the individual firm's  $q$  in Ghironi (1999a), it is possible to show that  $q_t^i = \hat{q}_t = q_t$ , where  $\hat{q}_t$  is measured using aggregate data, and  $q_t$  is defined in terms of detrended aggregate per capita variables:

$$q_t = \left\{ v_t + \sum_{s=t+1}^{\infty} R_{t,s} [(1+n)(1+g)]^{s-t} \left[ \frac{1}{\Psi_s} - (1-\tau_s^F) \right] y_s \right\} / [(1+n)(1+g)k_{t+1}],$$

$v_t \equiv V_t/E_t P_t$  is the detrended aggregate per capita equity value of the home economy; the discount factor  $R_{t,s}$  is defined by  $R_{t,s} \equiv 1 / \prod_{u=t+1}^s (1+r_u)$  and  $R_{t,t}$  is interpreted as 1;  $\Psi$  is the

markup charged by firms over marginal costs; and  $\tau^F$  is the rate of taxation of firms' revenues. I now define average  $q$  and the "adjustment for monopoly power" as:

$$q_t^{AVG} \equiv \frac{v_t}{(1+n)(1+g)k_{t+1}}, \quad adj_t \equiv \frac{\sum_{s=t+1}^{\infty} R_{t,s} [(1+n)(1+g)]^{s-t} \left[ \frac{1}{\Psi_s} - (1-\tau_s^F) \right] y_s}{(1+n)(1+g)k_{t+1}}. \quad (3.5)$$

When substitutability across goods is perfect,  $\Psi$  reduces to  $1/(1-\tau^F)$ , and marginal and average  $q$  coincide—the familiar Hayashi (1982) result. Measuring average  $q$  does not pose significant problems. Quarterly data for aggregate equity and firms' net capital assets since 1980:1 are available for the Canadian economy, and—together with the series for the CPI—they make it possible to obtain the desired measure.  $n$  is calculated as the average rate of growth of the Canadian population over the sample.

<sup>8</sup> A tilde over an arial rate denotes the percentage deviation of the *gross* rate from its steady-state level.

Measuring the adjustment term is harder. First, in reality agents do not have perfect foresight.  $adj_t$  is actually an expectation conditional on the information set available to firms at time  $t$  of the present discounted value of the “monopoly effect” from  $t + 1$  on. Second, this effect depends on the behavior of the markup, which itself needs to be estimated.

The equilibrium value of the markup can be written as  $\Psi_t = (1 - \gamma)y_t / (w_t L_t)$ . Given the estimate of the elasticity of GDP with respect to hours obtained above, I can calculate a baseline series of the markup using this expression. The results suggest that the Canadian economy is characterized by a fairly small degree of monopoly power at the aggregate level. The average level of the markup implied by the data is 1.11. Figure 1 shows the series of the growth rate of the markup over the sample period and the series of the growth rate of hours. As expected, and consistent with the evidence for the U.S. economy in Rotemberg and Woodford (1990), the markup is strongly countercyclical. This is a feature of the model that is important in the analysis of the model’s dynamics. Because the steady-state markup is given by  $\bar{\Psi}_0 = \theta / [(\theta - 1)(1 - \bar{\tau}_0^F)]$ , where  $\bar{\tau}_0^F$  is the unconditional mean of the series of the tax rate on firms’ revenues, it is possible to use this result to obtain an initial estimate of  $\theta$  around 12.08.<sup>9</sup>

I now define the variable  $\Omega_{t,s} \equiv R_{t,s} [(1+n)(1+g)]^{s-t} [(1/\Psi_s) - (1 - \tau_s^F)] y_s$ . I measure the relevant real interest rate with a series of the *ex post* real rate on Canadian T-Bills, calculated by deflating the nominal rate with CPI inflation.<sup>10</sup> As Figure 2 shows, if the series of the variable  $\Omega_{t,s}$  is plotted, the diagram suggests that the process for  $\Omega_{t,s}$  is non-stationary: the variance of the series drops to zero as time goes by.<sup>11</sup> When discounted back to the initial date by making use of the real interest rate, the variable  $X_s \equiv [(1+n)(1+g)]^{s-t} [(1/\Psi_s) - (1 - \tau_s^F)] y_s$  decays towards zero, *i.e.*, discounting by the real interest rate introduces a trend in  $\Omega_{t,s}$ .

It is possible to show that  $(1 + r_s)\Omega_{t,s} = (X_s / X_{s-1})\Omega_{t,s-1} = (1 + g_s^X)\Omega_{t,s-1}$ ,  $\forall s \geq t + 1$ , where  $g_s^X \equiv (X_s / X_{s-1}) - 1$ . Averaging the ratio  $X_s / X_{s-1}$  over the sample, it can be argued that the following process is a reasonable approximation for the behavior of  $\Omega_{t,s}$  in a stochastic setting:  $\Omega_{t,s} = [(1 + g^X) / (1 + r)]\omega \Omega_{t,s-1} + z_s$ , where  $z$  is a series of unanticipated disturbances. Writing the process for  $\Omega_{t,s}$  in this form makes it possible to remove the trending effect of the real interest rate when estimating the coefficient  $\omega$ . Running the autoregression and controlling for seasonal effects yields a highly significant estimate for  $\omega$  around .66.<sup>12</sup> The implied value of

<sup>9</sup> Although  $\bar{\tau}_0^F$  is assumed to be zero in most of the discussion, I do make use of the data on taxation of firms’ revenues in constructing a measure for  $adj$ . In particular, the rate of taxation of firms’ revenues is proxied by a series of the ratio of corporate income taxes to sales. Schmitt-Grohé (1998) calibrates the steady-state markup to 1.4 in her analysis of the transmission of U.S. business cycles to the Canadian economy. This yields an estimate for  $\theta$  of 3.68. To allow an easier comparison of my results with Schmitt-Grohé’s, I use this as baseline value of  $\theta$  in Ghironi (1999b) and in the example of Section 4.

<sup>10</sup> The series of the U.S. and Canadian real interest rates show an average differential of about 2 percentage points over the sample period—with the Canadian rate being higher than the U.S. on average. This contradicts real interest rate equalization not only in the short run but also over a fairly long horizon. For this reason, I use the Canadian real rate to calculate the steady-state real rate in Canada.

<sup>11</sup>  $s = 1980:1$  in the diagram.

<sup>12</sup> The standard error of the estimate is .047.

$[(1+g^x)/(1+r)]\omega$  is .73. Because the effects of  $g$  and  $n$  are already taken into account when detrending GDP in the definition of  $\Omega_{t,s}$ , one can also run the regression:

$$\Omega_{t,s} = [\omega'/(1+r)]\Omega_{t,s-1} + z_s. \quad (3.6)$$

The estimate for  $\omega'$  when seasonal effects are accounted for is around .79. The implied value of  $\omega'/(1+r)$  is .73, as expected.

Under the assumption that (3.6) is a reasonable approximation of the process for  $\Omega_{t,s}$ , the expectation of the realization of the process at any future date  $t+s$  is  $\tilde{E}_t\Omega_{t,s} = [\omega'/(1+r)]^{s-t}\Omega_{t,t}$ , where I differentiate the rational expectation operator—which I had not introduced so far—from the trend labor efficiency by use of a tilde. Thus:

$$adj_t = \frac{\Omega_{t,t} \sum_{s=t+1}^{\infty} [\omega'/(1+r)]^{s-t}}{(1+n)(1+g)k_{t+1}} = \frac{\omega'}{(1+r-\omega')(1+n)(1+g)} \left[ \frac{w_t L_t}{(1-\gamma)k_{t+1}} - (1-\tau_t^F) \frac{y_t}{k_{t+1}} \right]. \quad (3.7)$$

If the (exact) expression for  $adj$  in (3.5) is used to calculate its steady-state level,

$$adj_0 = \frac{1}{1+r-(1+n)(1+g)} \left[ \frac{\bar{w}_0 \bar{L}_0}{(1-\gamma)\bar{k}_0} - (1-\bar{\tau}_0^F) \frac{\bar{y}_0}{\bar{k}_0} \right].$$

The value of this expression should be close to that implied by the approximation in (3.7):

$$adj_0 = \frac{\omega'}{(1+r-\omega')(1+n)(1+g)} \left[ \frac{\bar{w}_0 \bar{L}_0}{(1-\gamma)\bar{k}_0} - (1-\bar{\tau}_0^F) \frac{\bar{y}_0}{\bar{k}_0} \right].$$

Thus, the model yields a theoretical value of  $\omega'$  approximately equal to  $(1+n)(1+g)$ . The data imply  $(1+n)(1+g) \cong 1.004$ . Hence, the estimate of  $\omega'$  falls short of the value that the theory would dictate by approximately .25. This notwithstanding, I will use (3.7) as my measure of the adjustment for monopoly power in the expression for marginal  $q$ , setting  $\omega'/(1+r-\omega') = 2.7$ .

Because the value of  $\omega'/[(1+r-\omega')(1+n)(1+g)]$  is but a normalization of the variable  $[(1/\Psi_t) - (1-\tau_t^F)]y_t/k_{t+1}$ , my choice does not affect the results of regressions in which average  $q$  and the adjustment for monopoly power are treated as separate variables.<sup>13</sup>

The series for average  $q$  and the adjustment effect calculated following the procedure described above suggest a fairly strong relation for the larger part of the sample between average  $q$  and investment if inventories are not included in the definition of investment and capital. The relation is somewhat weaker when inventories are included (see Figure 3).<sup>14</sup> The series for the

<sup>13</sup> Results obtained by selecting a much higher value for  $\omega'/(1+r-\omega')$  and using marginal  $q$  as a regressor were not significantly different. Note that I am implicitly assuming that the same process dictates the behavior of  $\Omega_{t+1,s}, \Omega_{t+2,s}, \dots$  and so on when firms are looking forward to formulate expectations about the behavior of output, taxation, and the markup at time  $t+1, t+2, \dots$  and so on. This is a strong assumption—which I will make again below—although it seems a reasonable one under normal economic conditions.

<sup>14</sup> Because I did not differentiate between capital accumulation in the form of fixed capital and accumulation of inventories, I define investment as the sum of the change in the fixed capital stock and in inventories. Analogously, my measure of the capital stock includes fixed capital and inventories. The data on stocks come from quarterly balance sheets for all industries. They are interpreted here as measures of end-of-period stocks. Thus, the data on

monopoly effect (not shown) is consistently negative—as suggested by familiar  $q$  theory—and shows a much larger volatility.

To gain a sense of the empirical performance of my measure for the economy-wide marginal  $q$ , I ran an initial regression of  $inv_t - k_t$  on  $q_t$ . This yielded a small and negative coefficient, in contrast with the theory. The values of  $R^2$  and the Durbin-Watson statistic signaled a very poor performance of the regression.<sup>15</sup> I separated the effects of average  $q$  and the monopoly adjustment factor on the investment-capital ratio by running a regression of  $inv_t - k_t$  on the series of the percentage deviations of average  $q$  and the monopoly adjustment factor from their steady-state levels. Because  $adj$  is negative, a positive deviation from the average signals a smaller monopolistic distortion, which increases marginal  $q$  and should cause larger investment.<sup>16</sup> The results of this regression were not encouraging either. The most likely explanation for the poor result of both regressions appeared to be serial correlation of the residuals. A regression in first differences proved more successful. The coefficient on average  $q$  was positive and significant. The coefficient on the monopoly factor turned out to be negative, but insignificantly different from zero. This suggested that the monopolistic distortion may not be a very relevant factor in driving the behavior of Canadian investment. It prompted me to continue treating average  $q$  and the adjustment for monopoly power as separate variables.<sup>17</sup>

Given log-linear equations for the investment part of the economy, I turn to estimating the rate of depreciation  $\delta$  and the parameter that dictates the size of the cost of adjusting the capital stock,  $\eta$ . I used non-linear least squares to estimate  $\delta$  from a regression based on (3.3):

$$k_t = Ak_{t-1} + \frac{(1+n)(1+g) - (1-\delta)}{(1+n)(1+g)} (inv_{t-1} - k_{t-1}).$$

Restricting  $A$  to be equal to 1 and omitting seasonal dummies yielded an estimate of  $\delta$  of .031.<sup>18</sup> Allowing  $A$  to differ from 1 and controlling for seasonal effects raised the estimate of the rate of depreciation to approximately .04.<sup>19</sup> I will use  $\delta = .035$  as baseline value in what follows.

Estimating  $\eta$  is more troublesome. Non-linear least squares regressions based on (3.2) ran into singularity problems, regardless of the separation of average  $q$  from the monopolistic distortion effect, and even when the latter was dropped. An alternative log-linearization of the

$k_t$  is the actual empirical correspondent of  $k_{t+1}$  in the model. Arguably, the behavior of inventories is ruled by a different model than that of fixed capital. Ramey and West (1997) survey the research on inventories. They argue that this variable should receive independent attention in analyses of business cycle fluctuations. They also provide arguments for the importance of inventories as a factor of production. Treating inventories as a productive factor in the empirical implementation of my model in the same way as I treat fixed capital seems consistent with Rotemberg and Woodford's (1993) argument about the importance of materials in production.

<sup>15</sup>  $R^2 = .027$ ,  $DW = .21$ .

<sup>16</sup> When the average of a series is negative, I calculate the percentage deviation of the series from the steady state as  $x_t = (x_t - \bar{x}_0) / |\bar{x}_0|$ .

<sup>17</sup> Schaller (1993) investigates the empirical performance of the  $q$  model for the Canadian economy using data from a panel of firms. He uses these data to construct series for  $q$  and shows that informational asymmetries cause firms' cash flows to have a significant impact on investment. Notwithstanding this result, I stick to the standard  $q$  model as an initial way to bring investment into the scene. Allowing for a role of cash flow in investment decisions is another direction along which the model can be extended.

<sup>18</sup> The standard error was .01,  $R^2 = .89$ .

<sup>19</sup> The estimate of  $A$  was close to .88, with a standard error around .04. The standard error for the estimate of  $\delta$  was .098 ( $R^2 = .9$ ).

investment equation, in which the steady-state relation between investment, capital, and  $q$  was not imposed, did not help.<sup>20</sup> Thus, in order to obtain a baseline value for  $\eta$ , I used the following approach. I ran an OLS regression in first differences of the investment-capital ratio on capital, average  $q$ , the monopoly adjustment factor, and a set of seasonal dummies. The coefficient on average  $q$  is approximately equal to  $(\bar{k}_0 \bar{q}_0^{AVG}) / (\eta \bar{i} \bar{n} v_0)$ . Given the estimated coefficient on average  $q$  in the regression, it is thus possible to obtain an approximate estimate for  $\eta$ . The procedure suggested that values of  $\eta$  as high as 2 were consistent with the estimated coefficient on average  $q$  in equations that included lagged capital as a regressor. This estimate implies that the cost of adjusting the capital stock is approximately equal to the ratio  $I^2/K$ —a very large amount. Mendoza (1991) uses annual data between 1946 and 1985 to calibrate a model of the Canadian economy. The cost of adjusting capital in his paper is  $(\eta/2)(I - \delta K)^2$ . He finds that values of  $\eta$  between .023 and .028 allow the model to mimic the observed percentage standard deviation of investment. The expression of the adjustment cost in my model allows the cost to decrease with firms' size and accounts for the fact that replacing depreciated capital can be as costly as a net addition to the capital stock. Leaving other reasons aside, the much larger value of  $\eta$  that is produced by my procedure can be at least partially explained by the different data frequency. A much larger adjustment cost would explain the very small changes in the capital stock that are observed on a quarterly basis.<sup>21</sup>

From firms' pricing,

$$\tilde{\pi}_t^{PPI} = \psi_t - \psi_{t-1} + \tilde{\pi}_t^{CPI} + l_t - l_{t-1}, \quad (3.8)$$

where  $\psi_t \equiv d\Psi_t / \bar{\Psi}_0$ ,  $\tilde{\pi}_t^{PPI}$  is producer price inflation,  $\tilde{\pi}_t^{CPI}$  is consumer price inflation, and  $l_t \equiv d \lambda_t / \bar{\lambda}_0$ , the latter being the shadow value of an additional unit of output. Log-

differentiating the expression of the markup in Ghironi (1999a) and substituting in (3.8) yields:

$$\tilde{\pi}_t^{PPI} = \tilde{\pi}_t^{CPI} + l_t - l_{t-1} - \frac{\phi(1 + \bar{\pi}_0)^2}{\theta - 1} \frac{\bar{k}_0}{\bar{y}_0} \left[ \tilde{\pi}_t^{PPI} - \tilde{\pi}_{t-1}^{PPI} - \frac{(1+n)(1+g)}{1+r} (\tilde{\pi}_{t+1}^{PPI} - \tilde{\pi}_t^{PPI}) \right].$$

$\phi$  measures the size of the cost of adjusting prices;  $\bar{\pi}_0$  is the steady-state level of PPI and CPI inflation;  $\theta > 1$  is the elasticity of substitution across goods. The markup reacts endogenously to the behavior of PPI inflation over time. Markup growth is smaller if the current growth in PPI inflation is larger. Faster PPI inflation growth has an unfavorable effect on output demand via its impact on relative prices. Firms sustain their profitability by slowing down growth in the markup component of prices. However, the change in the markup is larger if the future change in PPI inflation is expected to be large. This reflects firms' incentives to smooth the behavior of output price over time.

<sup>20</sup> It is:  $inv_t - k_t = \left\{ \left[ (\bar{q}_0 - 1) \bar{k}_0 / (\eta \bar{i} \bar{n} v_0) \right] - 1 \right\} k_t + \left[ \bar{q}_0 \bar{k}_0 / (\eta \bar{i} \bar{n} v_0) \right] q_t$ . The theory predicts that the steady-state levels of  $q$ ,  $k$ , and  $inv$  should be such that the first term in this equation is zero. However, the sample means of these variables suggest that this is not the case for realistic values of  $\eta$ .

<sup>21</sup> Bergin (1997) uses a model of investment similar to mine. He argues that a value of  $\eta$  as high as 20 would be required in a calibration of the model for it to generate results that are consistent with the empirical evidence on adjustment costs for Japanese firms reported by Hayashi and Inoue (1991). If compared to such value, my choice of 2 for the calibration exercise of Ghironi (1999b) and Section 4 appears conservative!

If labor demand and market clearing are taken into account, we have an equation for the dynamics of PPI inflation:

$$\tilde{\pi}_t^{PPI} - \tilde{\pi}_{t-1}^{PPI} = \frac{(1+n)(1+g)}{1+r} (\tilde{\pi}_{t+1}^{PPI} - \tilde{\pi}_t^{PPI}) + \frac{\theta-1}{\phi(1+\bar{\pi}_0)^2} \frac{\bar{y}_0}{k_0} \left[ w_t - w_{t-1} + L_t - L_{t-1} + \right. \\ \left. + (\theta-1)(\tilde{\pi}_t^{PPI} - \tilde{\pi}_t^{CPI}) - (y_t^W - y_{t-1}^W) \right]. \quad (3.9)$$

Today's acceleration in producer price inflation is faster the faster the future expected change in the inflation rate, the larger the real wage bill, and the larger the current deviation of PPI from CPI inflation. Instead, an increase in world output— $y^W$ —causes PPI inflation to slow down.

I tried to estimate  $\phi$  by running non-linear regressions based on equation (3.9). When the regressions did not run into singularity problems, the estimates of  $\phi$  turned out to be characterized by extremely large standard errors. Hence, I decided to limit myself to calibration of this parameter. If (3.9) is used, together with values of  $\theta$  between 3.68 and 12.08,  $\phi$  as high as 200 is required to generate a pattern of deviations of PPI inflation from the steady state that matches the behavior of the observed series. Would  $\phi = 200$  be an absurd value? The cost of adjusting prices is  $(\phi/2)(\pi^{PPI} - \bar{\pi}_0)^2 K$ . If steady-state quarterly inflation is about 1 percent per quarter, this says that increasing inflation by 10 percent—to 1.1 percent—would require the representative firm to purchase materials in an amount equal to .01 percent of its capital stock! Although the value of  $\phi$  is very high, the actual cost borne by the firm for a substantial acceleration in its output price inflation does not seem unrealistic.

Aggregate labor demand per capita can be written as:

$$L_t = -w_t - (\theta-1)RP_t - \psi_t + y_t^W, \quad (3.10)$$

Labor demand is larger if world output is larger. It is lower the higher the real wage and the markup. A higher real wage implies that the cost of labor is higher. A larger markup means that firms are exploiting their monopoly power more significantly. As a consequence, they supply less output and demand less labor. This is consistent with the empirical evidence on markup behavior in Rotemberg and Woodford (1990) and with the results obtained above. Larger deviations of the PPI from the CPI depress output demand more than they raise firms' profits for given output. Hence, they cause labor demand to decrease.

Because the equilibrium markup can be written as  $\Psi_t = (1-\gamma)y_t/w_t L_t$ , log-linearizing this expression and making use of the production function yields:

$$L_t = -[\theta/(1-\gamma)]RP_t - [\gamma/(1-\gamma)]k_t + [1/(1-\gamma)]y_t^W - [1/(1-\gamma)]Z_t. \quad (3.11)$$

The goods' market clearing condition ultimately determines labor demand in a small open economy. Given output demand, labor demand will be smaller if the capital stock is larger or if firms are experiencing favorable shocks to productivity.

Using the production function and the log-linear expression for the equilibrium markup makes it possible to show that increases in the markup, the real wage, and/or the labor-capital ratio cause the PPI to increase relative to the CPI:  $RP_t = \psi_t + w_t + \gamma(L_t - k_t) - Z_t$ . This can be combined with the definition of  $RP$  to obtain an alternative equation for PPI inflation that shows the direct linkage between the behavior of the PPI and that of the CPI in the model:

$$\tilde{\pi}_t^{PPI} = \tilde{\pi}_t^{CPI} + \psi_t - \psi_{t-1} + w_t - w_{t-1} + \gamma[L_t - L_{t-1} - (k_t - k_{t-1})] - (Z_t - Z_{t-1}). \quad (3.12)$$

Equation (3.12) is at the core of the results in Ghironi (1999b). Different monetary rules generate different CPI inflation dynamics. These cause different PPI dynamics, and thus different paths for the markup and the relative price of the representative domestic good. In turn, different markup and relative price dynamics affect the real side of the economy by changing firms' labor demand and investment decisions.

To verify the reliability of the estimates of the structural parameters obtained in this subsection, I ran full information maximum likelihood regressions of the system of equations that govern the production side of the economy. I took the following as starting values for the procedure:  $\delta = .035$ ,  $\phi = 200$ ,  $\gamma = .1$ ,  $\eta = 2$ ,  $\theta = 3.68$ . The estimated parameters differed somewhat, but the results generally supported my choice of these values as baseline for the empirical evaluation of alternative monetary rules for Canada in Ghironi (1999b).<sup>22</sup>

### 3.b. Consumption, Labor Supply, and Money Demand

The law of motion for detrended aggregate per capita consumption in the home economy is:

$$\mathbf{c}_{t+1} = \frac{\beta^\sigma (1+r)^\sigma (1+g)^{(1-\rho)(1-\sigma)}}{(1+n)(1+g)} [\mathbf{c}_t + \sigma \tilde{r}_{t+1} + (1-\rho)(1-\sigma)(w_{t+1} - w_t)] + \left[ 1 - \frac{\beta^\sigma (1+r)^\sigma (1+g)^{(1-\rho)(1-\sigma)}}{(1+n)(1+g)} \right] \mathbf{c}_{t+1}^{t+1}. \quad (3.13)$$

$c_{t+1}^{t+1} \equiv C_{t+1}^{t+1}/E_{t+1}$  is detrended consumption by the representative dynasty born at time  $t+1$  in the first period of its life.<sup>23</sup>  $\beta$  is the discount factor in intertemporal utility,  $\sigma$  is the intertemporal elasticity of substitution in utility from consumption and leisure, and  $\rho$  measures the relative importance of consumption versus leisure in utility. *Ceteris paribus*, a positive change in the real interest rate causes future consumption to increase relative to current. An increase in the real wage between  $t$  and  $t+1$  has a positive impact on aggregate per capita consumption at time  $t+1$  relative to its level in period  $t$  if  $\sigma$  is smaller than 1. The existence/stability condition of a steady state with constant real wage and interest rate obtained in Ghironi (1999a) ensures that an increase in the time  $t+1$ -newborn household's consumption causes aggregate per capita consumption at  $t+1$  to increase.

In order to analyze the response of aggregate per capita variables to shocks, it is necessary to determine  $c_{t+1}^{t+1}$ —the response of a newborn dynasty's consumption to the path of the shocks—as a function of variables that are *not* indexed by the dynasty's date of birth. A newborn household's consumption is a forward-looking variable that depends on the present discounted value of the entire stream of the dynasty's resources. Making use of the individual budget constraint and of the optimality conditions for newborn dynasties at time  $t+1$ , it is possible to

show that  $c_{t+1}^{t+1} = \rho \Theta_{t+1}^{-1} inc_{t+1}$ , where  $inc_{t+1} \equiv \sum_{s=t+1}^{\infty} R_{t+1,s} (1+g)^{s-(t+1)} [(1-\tau_s^L)w_s - t_s]$ ,

$\Theta_{t+1} \equiv \sum_{s=t+1}^{\infty} \beta^{\sigma[s-(t+1)]} (R_{t+1,s})^{1-\sigma} (1+g)^{(1-\rho)(1-\sigma)[s-(t+1)]} \left\{ \frac{(1-\tau_{t+1}^L)w_{t+1}}{(1-\tau_s^L)w_s} \right\}^{(1-\rho)(\sigma-1)}$ , and

$t_s \equiv T_s/E_s$  is detrended aggregate per capita lump-sum taxes and transfers (under the assumption that the latter do not differ across vintages).  $inc$  is the net present discounted value of the

<sup>22</sup> Given that the estimates did not differ too much, but were characterized by larger standard errors, I chose to use the values obtained from the single-equation procedure.

<sup>23</sup> See Ghironi (1999a) for details.

representative dynasty's endowment of time in terms of the real wage.  $\Theta^{-1}$  can be interpreted as a time-varying propensity to consume out of the available resources. In terms of percentage deviations from the steady state,  $c_{t+1}^{t+1} = -\tilde{\Theta}_{t+1} + inc_{t+1}$ . Assuming that  $\rho$  is sufficiently high, a persistent increase in the real wage rate that lasts beyond  $t + 1$  and causes the present discounted value of a newborn household's resources to be higher has a positive impact on aggregate per capita consumption at  $t + 1$  by inducing the newborn household to consume more in the initial period of its life.

The present discounted value of a household's lifetime endowment of time in terms of the real wage is another variable for which a proxy needs to be found. Because an agent's endowment of time does not change across periods, I will retain the assumption made in Ghironi (1999a) that this is normalized to 1 in each period also in the empirical analysis. In a stochastic setting,  $inc_{t+1}$  is actually defined by the rational expectation conditional on information available at time  $t + 1$  of the stream of net real wage rates. Following the procedure used to find a measure for marginal  $q$ , I define  $\Gamma_{t+1,s} \equiv R_{t+1,s} (1 + g)^{s-(t+1)} \left[ (1 - \tau_s^L) w_s - t_s \right]$ . The behavior of  $\Gamma_{t+1,s}$  is similar to that of  $\Omega_{t,s}$ . Thus, one can reasonably approximate the expectation of  $\Gamma_{t+1,s}$  at future dates with  $\tilde{E}_{t+1} \Gamma_{t+1,s} = [v/(1+r)]^{s-(t+1)} \Gamma_{t+1,t+1}$ , where  $v/(1+r)$  is the coefficient in the process for  $\Gamma_{t+1,s}$ :  $\Gamma_{t+1,s} = [v/(1+r)] \Gamma_{t+1,s-1} + \hat{z}_s, \forall s > t + 1$ . When this approximation is used,  $inc_{t+1} = [(1+r)/(1+r-v)] \left[ (1 - \tau_{t+1}^L) w_{t+1} - t_{t+1} \right]$ , and log-linearization around a steady state with no taxes yields  $inc_{t+1} = w_{t+1}$ .

This result suggests that, if the elements of the summation that defines  $inc$  decay towards zero as the time-horizon becomes longer, the percentage deviation of the current level of the real wage from its average is a good measure of the deviation of the present discounted value of the lifetime stream of real wage rates from its steady-state level. This result has implications for the findings of the literature on the sensitivity of aggregate consumption to current income. In an overlapping generations framework such as that explored in this paper, the aggregate Euler equation for consumption requires an adjustment that reflects the impact of a newborn generation's consumption on aggregate consumption. Because aggregate consumption at  $t - 1$  does not reflect this, time  $t$  income may contain information that is relevant for the behavior of aggregate consumption at time  $t$ , in conflict with the basic random walk result of Hall (1978). This is true regardless of the presence of liquidity constraints or other imperfections in financial markets.<sup>24</sup>

The last variable for which a proxy needs to be found is the time-varying propensity to consume  $\Theta^{-1}$ . I follow again the now familiar strategy. Given the expression for  $\Theta$ , I define  $\Sigma_{t+1,s} \equiv \beta^{\sigma[s-(t+1)]} (R_{t+1,s})^{1-\sigma} (1 + g)^{(1-\rho)(1-\sigma)[s-(t+1)]} \left\{ (1 - \tau_{t+1}^L) w_{t+1} / [(1 - \tau_s^L) w_s] \right\}^{(1-\rho)(\sigma-1)}$ . Assuming that the behavior of  $\Sigma_{t+1,s}$  can be reasonably approximated by a non-stationary process analogous to those used above,  $\Theta_{t+1}$  reduces to:  $\Theta_{t+1} = [(1+r)/(1+r-\xi)] \Sigma_{t+1,t+1} = (1+r)/(1+r-\xi)$ , where  $\xi$  would be the parameter to be estimated in the process. Because this is a constant, its impact is lost when the equation for  $c_{t+1}^{t+1}$  is log-linearized, leaving the deviation of this variable from its steady-state level depending only on  $inc_{t+1}$ .

<sup>24</sup> Of course, the importance of this phenomenon will be limited by the rate at which population is growing.

Log-linearization of the aggregate per capita labor-leisure tradeoff around a steady state with no taxes yields:

$$L_t = \left[ (1 - \rho) \bar{c}_0 / (\rho \bar{w}_0 \bar{L}_0) \right] (w_t - c_t). \quad (3.14)$$

Labor supply is an increasing function of the current real wage and a decreasing function of consumption. The latter is higher the higher the present discounted value of wage income. If agents expect to receive high wages in the future, their incentive to supply labor today is correspondingly weaker.

Equation (3.13) governs the intertemporal dynamics of aggregate consumption per capita. The intratemporal tradeoff between consumption and leisure at each point in time obeys equation (3.14). The representative dynasty's consumption Euler equation and labor-leisure tradeoff can be combined to obtain an Euler equation for labor supply. My approach to the estimation of  $\sigma$  and  $\rho$  relies on (3.13) and (3.14) as well as on the Euler equation for labor supply. In aggregate per capita terms, this can be written as:

$$L_{t+1} = \frac{\beta^\sigma (1+r)^\sigma (1+g)^{(1-\rho)(1-\sigma)}}{(1+n)(1+g)} \left\{ L_t - \frac{1-\rho}{\rho} \frac{\bar{c}_0}{\bar{w}_0 \bar{L}_0} \left\{ \sigma \tilde{i}_{t+1} - [1 - (1-\rho)(1-\sigma)] (w_{t+1} - w_t) \right\} \right\}. \quad (3.15)$$

If  $\sigma > 1 - [1/(1-\rho)]$ , an increase in the detrended real wage between  $t$  and  $t+1$  causes future aggregate per capita leisure to decrease, *i.e.*, it causes labor supply to increase. Because  $c_{t+1}^{t+1} \equiv w_{t+1}$  under my assumptions, the independent effect of a newborn dynasty's labor-leisure choice on the aggregate supply of labor at  $t+1$  washes out.

From the aggregate per capita money demand equation, it is possible to obtain an equation for the rate of growth of detrended real money balances. In log-linear terms:

$$\tilde{g}_t^m = (\mu/\sigma)(c_t - c_{t-1}) - (\mu/\bar{i}_0)(\tilde{i}_{t+1} - \tilde{i}_t) - \mu(1-\rho)[(1-\sigma)/\sigma](w_t - w_{t-1}). \quad (3.16)$$

$\mu$  is the intertemporal elasticity of substitution in utility from real money balances. Faster consumption growth causes faster growth in real balances. Aggregate per capita money balances grow more slowly if the growth in the opportunity cost of holding money—the nominal interest rate  $\tilde{i}$ —is faster. *Ceteris paribus*, if  $\sigma < 1$ , faster growth in the wage rate causes growth in real balances to slow down.

Given the log-linear first-order conditions for consumption, labor supply, and money demand, it is possible to estimate the intertemporal elasticity of substitution in utility of consumption and leisure,  $\sigma$ ; the relative importance of consumption and leisure in utility,  $\rho$ ; and the intertemporal elasticity of substitution in demand for real money balances,  $\mu$ .

I begin with the estimation of  $\rho$ . A simple non-linear least squares regression of hours on the difference between the percentage deviations of the real wage and consumption from the steady state as in equation (3.14) yielded an estimate of  $\rho$  of 2.93.<sup>25</sup> One reason for the failure to obtain a value of  $\rho$  between 0 and 1 could be the nature of the data. I use a series of actual hours worked in the Canadian economy. This is more likely to reflect labor *demand* than labor *supply* in an economy where unemployment is an issue. A strong negative effect of the real wage on hours due to labor demand dynamics may cause the estimate of  $\rho$  to be larger than 1. To explore this possibility, I ran a simple unrestricted regression of hours on the real wage and consumption. The estimated coefficients were both positive. The coefficient on the real wage was small and hardly

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<sup>25</sup> Standard error = .86.

significant, but the coefficient on consumption was very significantly different from zero.<sup>26</sup> The result was thus at odds with the initial conjecture, and the reason appeared to be a *positive* impact of consumption on hours, rather than negative as predicted by the theory. I thus ran the following regression:<sup>27</sup>

$$L_t = \frac{1-\rho}{\rho} \frac{\bar{c}_0}{\bar{w}_0 \bar{L}_0} w_t - \left( \frac{1-\rho}{\rho} \frac{\bar{c}_0}{\bar{w}_0 \bar{L}_0} + A \right) c_t,$$

assuming an initial value of zero for the parameter  $A$ . This yielded an estimate for  $\rho$  of .79, with a standard error of .228. The estimate for  $A$  was -.94, with a standard error of .36. Allowing the coefficient of consumption in the labor-leisure tradeoff equation to differ from the prediction of the theory yielded a fairly high estimate of  $\rho$ —though smaller than 1, consistent with the expectation of a small coefficient for the real wage.<sup>28</sup>

To take care of the serial correlation in the residuals signaled by a low Durbin-Watson statistic, I ran:

$$L_t = A_1 L_{t-1} + \frac{1-\rho}{\rho} \frac{\bar{c}_0}{\bar{w}_0 \bar{L}_0} w_t - \left( \frac{1-\rho}{\rho} \frac{\bar{c}_0}{\bar{w}_0 \bar{L}_0} + A_2 \right) c_t,$$

with an initial value of zero for  $A_1$ . When only the significant seasonal dummies were included<sup>29</sup>, the estimates (standard errors) of  $\rho$ ,  $A_1$ , and  $A_2$  were, respectively, .62 (.081), .46 (.072), and -.96 (.2).<sup>30</sup>

The results of the regressions based on the labor leisure tradeoff thus suggested a range of values between .57 and .79 for  $\rho$ . By including lagged hours as an explanatory variable, the last regression somewhat shifted the focus to Euler-equation type considerations. I thus ran a second set of single-equation regressions based on equation (3.15) to verify if this yielded similar results. The first was an exploratory regression of hours on lagged hours, the real interest rate, and real wage growth. The coefficient on the real interest rate was positive but hardly significant. Hence, I tried to separate the effects of the nominal interest rate and inflation and ran:

$$L_t = A_1 L_{t-1} + A_2 \tilde{i}_t + A_3 \tilde{\pi}_t^{CPI} + A_4 (w_t - w_{t-1}).$$

The estimates were as follows (standard errors in parenthesis):

$A_1$ : .499 (.096);  $A_2$ : .4 (.21);  $A_3$ : -1.59 (.86);  $A_4$ : -1.52 (.47);  $R^2 = .40$ ,  $DW = 2.14$ .

The nominal interest rate and inflation had comparable levels of significance. When seasonal dummies were included, they were significant, and the estimated coefficients changed to:

$A_1$ : .83 (.069);  $A_2$ : .21 (.13);  $A_3$ : -1.13 (.53);  $A_4$ : -.177 (.43);  $R^2 = .8$ ,  $DW = 2.42$ .

These results suggested some preliminary observations. Contrary to the predictions of the theory, the impact of the real interest rate on the supply of hours appeared positive. Separating the nominal interest rate from inflation did not seem to change this result. Because higher inflation is consistent with a lower real interest rate, the theory would suggest that higher inflation causes labor supply to be higher, but this is not consistent with the finding of the exploratory regression.

<sup>26</sup> The coefficient on the real wage was .19 (standard error = .26, t-statistic = .74); the coefficient on consumption was .74 (.12, 5.98).  $R^2 = .43$ ,  $DW = 1.59$ .

<sup>27</sup> The error term is omitted.

<sup>28</sup> If I included seasonal dummies in the regression, the estimate of  $\rho$  dropped to .57 and that of  $A$  became -1.39. All coefficients were highly significant.

<sup>29</sup> The dummy for the third season turned out to be only marginally significant.

<sup>30</sup>  $R^2 = .84$ ,  $DW = 2.17$ .

Once seasonal dummies were included, the effect of real wage growth was not significantly different from zero.

I then imposed the parameter restrictions dictated by the model and ran the non-linear least squares regression:

$$L_t = \frac{\beta^\sigma (1+r)^\sigma (1+g)^{(1-\rho)(1-\sigma)}}{(1+n)(1+g)} \left\{ L_{t-1} - \frac{1-\rho}{\rho} \frac{\bar{c}_0}{\bar{w}_0 \bar{L}_0} \left\{ \sigma \tilde{r}_t - [1 - (1-\rho)(1-\sigma)](w_t - w_{t-1}) \right\} \right\}.$$

I calibrated  $\beta$  to .99—a fairly safe choice for the discount factor at quarterly frequency. I did a grid search over a range of values of  $\sigma$  between .01 and .31 (this choice will be motivated below) and found an estimate of  $\rho$  consistently above 1. This result seemed to support the observation that the effect of the real interest rate can be positive—as suggested by the regressions above—only if  $\rho$  is larger than 1. However, the result vanished once I controlled for seasonal effects. For  $\sigma = .16$ , the estimated value of  $\rho$  turned out to be .695 (.317). The estimate raised to .74 for  $\sigma = .21$ , with a slight decrease in the value of the likelihood function (from 173.657 to 173.510).<sup>31</sup>

I tried GMM and IV estimation to control for correlation of variables with the error term and endogeneity effects, always including seasonal dummies in the regressions. I used lagged hours, real interest rate, and wage as instruments. Non-linear IV yielded estimates of  $\rho$  significantly above 1 for very low values of  $\sigma$ , but the estimates were close to those obtained with the non-linear least squares regression for  $\sigma$  between .11 and .31, although with larger standard errors. GMM estimation with the same set of instruments and with starting values via non-linear two-stage least squares yielded values of  $\rho$  greater than 1 over a larger range of values of  $\sigma$ . For  $\sigma = .21$ , the estimate of  $\rho$  was .55 (.16). The estimated  $\rho$  was somewhat lower when the starting value was not chosen via non-linear 2SLS.

To summarize, under the assumption that the series of actual hours worked does contain information on labor supply behavior, the results of the single-equation regressions based on both the intratemporal tradeoff equation and the intertemporal optimality condition for labor supply suggest a range of values for  $\rho$  between .55 and .8. .55 seems too low a weight for consumption in agents' utility. The regressions below will actually suggest that values of  $\rho$  as high as .99 cannot be dismissed.

The consumption Euler equation can be used to obtain an estimate of  $\sigma$ . I initially tried a non-linear least squares regression based on the following equation, doing grid searches over a range of values of  $\rho$ :

$$c_t = \frac{\beta^\sigma (1+r)^\sigma (1+g)^{(1-\rho)(1-\sigma)}}{(1+n)(1+g)} [c_{t-1} + \sigma \tilde{r}_t + (1-\rho)(1-\sigma)(w_t - w_{t-1})] + \left[ 1 - \frac{\beta^\sigma (1+r)^\sigma (1+g)^{(1-\rho)(1-\sigma)}}{(1+n)(1+g)} \right] w_t. \quad (3.17)$$

The regression yielded *negative* and significant estimates for  $\sigma$ , with the likelihood function increasing for higher values of  $\rho$ . This result appeared puzzling.<sup>32</sup> In order to gain an understanding of what could motivate it, I ran an unrestricted OLS regression of the type:

$$c_t = A_1 c_{t-1} + A_2 \tilde{r}_t + A_3 w_t + A_4 (w_t - w_{t-1}) + \sum_{i=5}^8 A_i D_i,$$

<sup>31</sup> Standard error = .286.

<sup>32</sup> The result was robust to alternative estimation techniques (GMM, IV) and to the use of U.S. variables rather than lagged Canadian ones as instruments.

where the  $D_t$ s are seasonal dummies. The estimates were as follows:

$A_1$ : 1.04 (.038);  $A_2$ : -.18 (.047);  $A_3$ : .28 (.098);  $A_4$ : -.099 (.17);  $R^2 = .96$ ,  $DW = 2.17$ .

Contrary to what the theory would suggest, the real interest rate has a negative and significant impact on consumption. This result resembles the findings of a previous exploration of this type of models for the Canadian economy by Altonji and Ham (1990). The coefficient on real wage growth appears insignificantly different from zero. This is consistent with a value of  $\rho$  close to 1 in equation (3.17). The current real wage has a positive and significant effect on consumption.<sup>33</sup> The fact that the real interest rate has a negative impact on consumption explains the negative estimate of  $\sigma$  in the initial non-linear least squares regression. The coefficient on the real interest rate is equal to  $\sigma$  times the coefficient on lagged consumption. Because the latter is positive, the negative effect of the real interest rate on consumption translates into a negative estimate of  $\sigma$ .

Following Altonji and Ham (1990), I separated the impact of the nominal interest rate and inflation and ran the exploratory regression:

$$c_t = A_1 c_{t-1} + A_2 \tilde{i}_t + A_3 \tilde{\pi}_t^{CPI} + A_4 w_t .$$

Based on the previous results, I dropped the real wage growth term. The results were:

$A_1$ : .99 (.036);  $A_2$ : -.07 (.056);  $A_3$ : -.35 (.21);  $A_4$ : .19 (.087);  $R^2 = .96$ ,  $DW = 1.94$ .

The impact of both the nominal interest rate and inflation on consumption is only marginally significant, though the effect of inflation is larger. This motivates the negative effect of the real interest rate.

The result of the previous regression induced me to drop the nominal interest rate from the regressions used to estimate  $\sigma$  and to focus on the effect of inflation. To get an initial estimate of  $\sigma$ , I ran:

$$c_t = A c_{t-1} - \sigma A \tilde{\pi}_t^{CPI} + (1 - A) w_t .$$

I found  $A = .95$  (.02) and  $\sigma = .55$  (.14).<sup>34</sup> This was a more encouraging result, although the value of the elasticity was significantly higher than that found by Altonji and Ham (1990). I then went back to the log-linear Euler equation and ran the non-linear least squares regression:

$$c_t = \frac{\beta^\sigma (1+r)^\sigma (1+g)^{(1-\rho)(1-\sigma)}}{(1+n)(1+g)} (c_{t-1} - \sigma \tilde{\pi}_t^{CPI}) + \left[ 1 - \frac{\beta^\sigma (1+r)^\sigma (1+g)^{(1-\rho)(1-\sigma)}}{(1+n)(1+g)} \right] w_t .$$

$\beta$  and  $\rho$  were set to .99. I chose such a high value of  $\rho$  for consistency with the statistical insignificance of the coefficient on real wage growth.<sup>35</sup> The estimated intertemporal elasticity of substitution was .23, with a standard error of .1 ( $R^2 = .95$ ,  $DW = 1.64$ ). Reintroducing the real wage growth term in the regression did not affect the results significantly. The coefficients on seasonal dummies turned out to be insignificantly different from zero.

In order to take care of problems of correlation between the error term and the regressors and of issues of endogeneity, I re-estimated  $\sigma$  using non-linear IV and GMM. Following the suggestion of Altonji and Ham (1990), I tried alternative sets of instruments—lagged Canadian

<sup>33</sup> As suggested above, this result—which conflicts with the basic random walk hypothesis—is consistent with the dynamics of population in the economy and needs no imperfection in capital markets to be explained. However, the low rate of population growth in Canada implies that imperfections in financial markets are likely to be a more relevant empirical motivation of the result.

<sup>34</sup>  $R^2 = .96$ ,  $DW = 1.67$ . When I added seasonal dummies, they were not significant.

<sup>35</sup> The results of the regressions below suggest that the coefficient is not statistically insignificant because of a value of  $\sigma$  close to 1.

variables and U.S. variables. A GMM estimation with starting values via non-linear 2SLS and lagged Canadian consumption, CPI inflation, and real wage as instruments yielded a value for  $\sigma$  of .14 (.11). Setting  $\sigma = 1$  as starting value raised the estimate to .18, with approximately the same standard error.<sup>36</sup> When I used U.S. GDP and inflation as instruments, I found a higher value of  $\sigma$ , around .21. Overall, these single-equation regressions seem to support a range of values between .14 and .25 as plausible for the intertemporal elasticity of substitution in the Canadian economy. Altonji and Ham (1990) concluded in favor of a range between .1 and .2 using annual data between 1951 and 1984. My results seem fairly consistent with their findings.

The next structural parameter to be estimated is the intertemporal elasticity of substitution in utility from real money balances— $\mu$ . I initially ran the exploratory regression:

$$\tilde{g}_t^m = A_1(c_t - c_{t-1}) + A_2(\tilde{i}_{t+1} - \tilde{i}_t) + A_3(w_t - w_{t-1}).$$

Because  $i_{t+1}$  is the nominal interest rate between  $t$  and  $t + 1$ , it is known by agents at time  $t$ . The estimated coefficients were:

$A_1$ : 2.01 (.63);  $A_2$ : - 1.09 (.51);  $A_3$ : .6 (.66);  $R^2 = .19$ ,  $DW = 3.24$ .

When taken into account, seasonal effects were small but significantly different from zero. The estimate of  $A_1$  dropped to 1.39 and that of  $A_2$  to - .93.  $R^2$  increased to .74 and—more importantly— $DW$  dropped to 2.2. The estimate of  $A_3$  was hardly significant. This could be interpreted as a further signal that a high value of  $\rho$  may be consistent with the data.

Given these results, I ran the following non-linear least squares regression:

$$\tilde{g}_t^m = (\mu/\sigma)(c_t - c_{t-1}) - (\mu/\bar{i}_0)(\tilde{i}_{t+1} - \tilde{i}_t) + \sum_{i=1}^4 A_i D_i,$$

doing a grid search over a range of values for  $\sigma$ .  $\bar{i}_0$  is the unconditional mean of the series of the T-Bills interest rate over the sample period. For  $\sigma$  as low as .01, the procedure yielded an estimate of  $\mu$  of .015 with a t-statistic of 3.8 ( $R^2 = .71$ ,  $DW = 2.11$ ). For a value of  $\sigma$  in the Altonji-Ham range (.11), the estimated value of  $\mu$  was .106, with a standard error of .024. The value of the log-likelihood was 151.089. When  $\sigma = .31$ , the estimated  $\mu$  was .099 (.029). The log-likelihood was 147.710. Reintroducing the real wage growth term in the regression did not affect the results significantly.

Thus, results from the single equation regressions suggest the following ranges for the relevant structural parameters:  $\mu \in [.09, .12]$ ,  $\rho \in [.8, .99]$ ,  $\sigma \in [.1, .25]$ . As I had done for the production side of the economy, I used these estimates as starting values for FIML regressions of the system of the consumers' first-order conditions, to verify if the procedure yielded similar estimates. The results of the FIML estimation were more reliable in this case: estimates came with small standard errors. The results confirmed the ranges above for  $\mu$ ,  $\rho$ , and  $\sigma$ . They suggested that values of  $\sigma$  smaller than .1 could be plausible, and that  $\mu$  could be as low as .07.

To summarize the results of this section, Table 1 displays a set of parameter values that were found to be plausible for the Canadian economy and are used in the policy rule evaluation exercise of Ghironi (1999b) and in the example below.

Given the sample mean of the Canadian T-Bills rate,  $\mu = .08$  ensures that the elasticity of growth in demand for real balances to the interest rate is not too large. I choose  $\sigma = .1$  for reasons

<sup>36</sup> The instruments were consumption and the real wage at dates  $t - 1$  and  $t - 2$  and inflation at  $t - 1$ . Using consumption and the real wage at  $t - 2$  and  $t - 3$  and inflation at  $t - 2$  as instruments yielded  $\sigma = .146$  (.11).

of convergence of aggregate consumption to the steady state after a shock.<sup>37</sup> In order to speed up convergence, I actually set  $\beta = .95$  rather than  $.99$  in the calibration exercise. This change in the value of  $\beta$  does not affect the ranges of estimates of the other parameters in any significant way and is useful for exposition purposes.  $\rho$  is probably a more controversial choice, though  $.8$  seems a sensible starting value.

#### 4. A Recession in the United States

In this section, I illustrate the functioning of the model by analyzing the transmission of a recession in the U.S. to Canada under inflation targeting—the monetary regime currently followed by the Bank of Canada.

Shocks to U.S. variables cannot be taken in isolation. In Ghironi (1999a), I do not model the structure of the U.S. economy as explicitly as Canada's but—at a minimum—one must recognize that four variables that appear in the equations for Canadian variables will be affected by shocks to U.S. GDP or interest rate: besides these, the U.S. CPI inflation rate and the real interest rate will change. Movements in U.S. inflation and interest rates will affect Canada through PPP and interest rate parities. One cannot analyze the consequences of a shock to U.S. output or the interest rate for Canada without explicitly accounting for the comovements in all relevant variables that are triggered by the initial disturbance.

In the exercise, I impose a minimal amount of structure on the U.S. economy. I take the Federal Funds Rate to be the relevant nominal interest rate. The Federal Reserve is assumed to set this rate as its policy instrument. Following Rotemberg and Woodford (1997), I assume that the Fed sets the nominal interest rate based on a reaction function that depends on past levels of the rate and on the current and past levels of CPI inflation and GDP. In terms of deviations from the steady state:<sup>38</sup>

$$\tilde{i}_{t+1}^* = \sum_{k=1}^{n_I^*} \varphi_{1k} \tilde{i}_{t+1-k}^* + \sum_{k=0}^{n_{\pi}^*} \varphi_{2k} \tilde{\pi}_{t-k}^{CPI^*} + \sum_{k=0}^{n_Y^W} \varphi_{3k} y_{t-k}^W. \quad (4.1)$$

Shocks to this equation are exogenous shocks to monetary policy. Because Canada is small relative to the U.S., the Fed's reaction function does not incorporate any Canadian variable. The negligible impact of Canadian GDP on world aggregates allows me to identify U.S. GDP with  $y^W$  in the model.

I model the U.S. economy as a recursive structural VAR that includes equation (4.1) and equations for GDP and inflation. The state vector is  $[\tilde{\pi}_t^{CPI^*}, y_t^W, \tilde{i}_{t+1}^*]'$ , and the causal ordering of variables is the order in which they are listed. I follow Rotemberg and Woodford (1997) in assuming that the interest rate affects output and inflation only with a lag, but I do not include

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<sup>37</sup> Higher values of  $\sigma$  yield slower convergence to the steady state. In addition, because—with one exception—I calibrate the exact equations of the model in the exercise, but the data suggest that the nominal interest rate may have a very small impact on consumption, I choose a value of  $\sigma$  that ensures such small effect. The exception is that I use the equation in footnote 20 for investment. If I did not do that, I would have an investment equation whose coefficients are absolutely out of line with what suggested by the regressions. In that case,  $\eta = 20$  would actually be required to yield sensible coefficients, but this would cause absurdly high adjustments costs (in the order of 70 percent of GDP during the sample period).

<sup>38</sup>  $\tilde{i}_{t+1}^*$  is the time  $t$  nominal interest rate.

future inflation and GDP in the time- $t$  state vector, because I do not believe that future consumption and inflation levels are entirely predetermined.

I estimate the VAR with three lags using full information maximum likelihood. I use data between 1980:1 and 1997:4. The estimated coefficients for the three equations and the standard errors are in the columns of Table 2. Seasonal dummies were not significant, as well as further lags. The estimated coefficients for the Fed's reaction function suggest behavior in line with a generalized Taylor rule, consistent with the findings of Rotemberg and Woodford (1997).

Figure 4 shows the responses of GDP, inflation, and the Federal Funds Rate to a 1% decrease in U.S. GDP.<sup>39 40</sup> The deviation of GDP from the steady state increases in the first two quarters. Inflation reacts with a lag, and subsequently drops. The Fed reacts immediately by lowering the Federal Funds Rate to sustain GDP.<sup>41</sup> Over time, all variables go back to the steady state.

The paths of U.S. variables generated by the shock constitute the paths of the exogenous world-economy variables following the initial impulse in my model of the Canadian economy. The estimated VAR equations are included in the system of equations that govern the dynamics of the world economy following the initial shock, along with the model equations for Canada and the monetary rule followed by the Bank of Canada. The system is then solved using routines that follow Uhlig (1997).

The monetary rule followed by the Bank of Canada is perfectly neutral as far as the real interest rate is concerned in all periods in which no shock happens. Under all rules the Canadian real interest rate is equal to the U.S. real interest rate—except in the case of short-run deviations from uncovered interest parity due to unexpected shocks. Different rules can make a difference for the dynamics of the Canadian economy via real interest rate effects only in the very short run. However, alternative policy rules can produce different dynamics by causing differences in the behavior of the relative price,  $p(i)/P$ , which can be taken as a measure of the terms of trade for the Canadian economy.<sup>42</sup>

In this illustrative example, I focus on inflation targeting, the regime currently followed by the Bank of Canada. Under this regime, I assume that the Bank of Canada sets the Canadian nominal interest rate to keep CPI inflation at its steady-state level in *all* periods, including when an unexpected shock happens:  $\tilde{\pi}_t^{CPI} = 0 \quad \forall t \geq t_0$ .<sup>43 44</sup>

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<sup>39</sup> Because markets clear in the model, an exogenous decrease in U.S. GDP can be interpreted both as the consequence of a generalized decline in world demand for goods and as the outcome of a negative supply shock. I interpret the shock as an exogenous contraction in demand. The interpretation is consistent with the fact that U.S. inflation declines following the disturbance.

<sup>40</sup> In the impulse responses, the level of the interest rate at each point in time is the value chosen by the monetary authority at that date.

<sup>41</sup> A measure of the U.S. real interest rate can be obtained by using the response of the inflation rate to deflate that of the Federal Funds Rate. The real interest rate reacts with a lag. It falls below the steady state in the first quarter after the shock and remains lower than its long-run level until it eventually returns to it.

<sup>42</sup> The terms of trade are actually given by  $p(i)/(\epsilon P^*(f))$ , where  $P^*(f)$  is the U.S. PPI. Under my assumptions, the fraction of Canadian goods in the U.S. consumption bundle is negligible. Hence,  $P^*(f)$  is only marginally different from  $P^*$ . Because of purchasing power parity,  $p(i)/(\epsilon P^*) = p(i)/P$ .

<sup>43</sup> Svensson (1998) distinguishes between strict and flexible inflation targeting depending on the weights attached to different targets in the policymaker's loss function. In both cases, the central bank is minimizing a loss function

An operational rule that is consistent with this target can be obtained from the money demand equation. Inflation will be constant at its steady-state level in all periods if the Bank of Canada sets its interest rate as:

$$\tilde{i}_{t+1} = \tilde{i}_t - (\bar{i}_0/\mu)\tilde{g}_t^M + (\bar{i}_0/\sigma)(c_t - c_{t-1}) - \bar{i}_0(1-\rho)[(1-\sigma)/\sigma](w_t - w_{t-1}).$$

Strict inflation targeting requires the Bank of Canada to raise its interest rate if it did so in the previous period and if consumption is growing, and to lower it if money growth and/or the real wage are rising.<sup>45</sup>

Impulse responses under this rule are in Figure 5. The Canadian dollar depreciates after the initial period, consistent with the decline in U.S. inflation.<sup>46</sup> The Bank of Canada lowers its interest rate by less than the Fed, which generates expectation of depreciation in the periods following the shock. PPI inflation falls slightly on impact, but climbs above the steady state by even less within a year. The relative price of the representative Canadian good falls, and the markup rises. According to the model, firms absorb part of the contractionary consequences of the shock by accepting a lower real price for their output. They preserve profitability by raising the markup component of the price level. Consistent with theory and data, the markup is countercyclical. Labor demand falls, and so does the real wage, though both are above the steady state six quarters after the shock. Tobin's  $q$  rises. Firms invest and substitute capital for labor.<sup>47</sup> This causes an initial increase in share price inflation, which is absorbed rapidly. GDP drops, but it bounces above the steady state in two years. The recession is not as pronounced as in the U.S. Consumption falls and, save for a brief recovery in the third and fourth quarters after the shock, it continues to fall for eight years. It goes back to the steady state only in the very long run.<sup>48</sup> Money balances initially rise, but fall below the steady state in the third quarter. Canada runs a fairly persistent current-account deficit.<sup>49</sup>

Schmitt-Grohé (1998) uses a VAR approach to study how shocks to U.S. GDP are transmitted to the Canadian economy. She compares the predictions of the estimated VAR to those of alternative flexible price models. She finds that the models can explain the observed responses of output, hours, and capital only if larger-than-realistic movements in relative prices are allowed. She raises the question of whether relaxing the assumption of price flexibility might

that attaches weight to inflation variability and interest-rate changes. Under flexible inflation targeting, the policymaker cares about output volatility too. In reality, the Bank of Canada has an interval target for CPI inflation.

<sup>44</sup> An important dimension of inflation targeting in the real world is the choice of the steady-state inflation rate, *i.e.*, the actual target, given by  $\bar{\pi}_0$  in my model. When evaluating the consequences of inflation targeting, one would want to discuss the implications of different levels of the target. My model is not appropriate to perform such analysis. Because firms face costs of output price inflation volatility around  $\bar{\pi}_0$ , the choice of the steady-state level of inflation has no consequence for welfare in the economy, because it does not affect the steady-state level of the markup or the dynamics of relative prices.

<sup>45</sup> The latter as long as  $\sigma < 1$ .

<sup>46</sup> I use the word depreciation to refer to an increase of the rate of depreciation above its steady-state level. The rate of depreciation does not change on impact because U.S. inflation reacts with a lag.

<sup>47</sup> In the impulse responses, capital at each point in time is capital at the end of the corresponding period rather than at the beginning.

<sup>48</sup> Slow population growth motivates slow convergence of consumption to the steady state.

<sup>49</sup> The large deviation of the current account from the steady state should not alarm the reader. Because the steady-state current-account deficit in Canada is approximately .59% of GDP, the deficit never exceeds 5% of GDP in the exercise, remaining within entirely plausible limits.

help remove the puzzle. My exercise provides a positive answer to the question. The impulse responses for output, hours, and capital in the example above are consistent with the size and duration of the recession in the U.S. At the same time, fairly small movements in the relative price of Canadian goods and other variables are observed.

In Ghironi (1999*b*), I compare inflation targeting to several other rules, including a fixed exchange rate regime with the U.S. and the Taylor rule. I assume that Canada is subject to three sources of volatility: shocks to U.S. GDP, exogenous movements in U.S. monetary policy, and technology shocks. Welfare comparisons support the choice of a constant inflation rate as the optimal monetary rule among those considered for the Canadian economy. Markup and relative price dynamics under that rule ensure the smallest volatility of consumption. Abandoning inflation targeting for a fixed exchange rate regime with the U.S. would not be welfare improving, although the performance of the two regimes is fairly similar. A forward-looking version of the Taylor rule dominates the traditional version.

## 5. Conclusions

I have estimated the structural parameters of a small open economy model using data from Canada and the United States. The model—presented in detail in Ghironi (1999*a*)—improves upon the existing theoretical literature from an empirical perspective. I solve the stationarity problem that characterizes several existing models by adopting an overlapping generations structure à la Weil (1989), in which new infinitely lived agents enter the economy at each point in time owning no assets. I model nominal rigidity explicitly, incorporating endogenous markup variability, and I bring investment and capital accumulation into the analysis. The paper also illustrated a plausible strategy for constructing measures for unobservable variables. I estimated the parameters mainly by making use of non-linear least squares at the single equation level. I then verified whether multiple-equation regressions yielded significantly different estimates by running full information maximum likelihood regressions based on the systems of the consumers' first-order conditions and the firms' first-order conditions. The approach was fairly successful. Non-linear least squares and full information maximum likelihood yielded fairly similar estimates. Most parameter estimates were characterized by small standard errors and were in line with the findings of other studies. Perhaps not surprisingly, given the use of aggregate data for the estimation procedure, the size of the nominal rigidity and the size of the cost of adjusting the capital stock were the only troublesome parameters. More work on these is to be done in the future.

In Section 4, I illustrated the functioning of the model by analyzing the transmission of a recession in the U.S. to Canada under inflation targeting, the monetary regime currently followed by the Bank of Canada. Consistent with the goal of combining theoretical rigor with an empirical approach, I used a simple VAR to trace the impact of comovements in U.S. variables on the Canadian economy. The estimated VAR equations for the U.S. were combined with the model equations for Canada to determine the response of the Canadian economy to the initial shock. The example illustrated the role of markup and relative price dynamics in the model.

In Ghironi (1999*b*), I use the model and the parameter estimates to evaluate the performance of alternative monetary rules for the Canadian economy. A constant path of inflation yields the smallest volatility of consumption and is thus the optimal rule among those I consider. The exercise provides interesting insights on the pros and cons of different rules, pointing to the importance of transmission channels that are not featured in other studies of monetary rules for

open economies. The findings of this paper—which I see as an initial contribution to “new open economy macroeconometrics”—and of its companion suggest an empirical “case-by-case” approach as a profitable way to understand macroeconomic interdependence and address relevant policy issues.

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**Table 1.** Structural parameters

$\beta = .95$	$\mu = .08$	$\rho = .8$	$\sigma = .1$	$\delta = .035$	$\phi = 200$
$\gamma = .1$	$\eta = 2$	$\theta = 3.68$	$n = .00308$	$g = .00134$	$r = .07988$

**Table 2.** The U.S. economy

	$\tilde{\pi}_t^{CPI*}$	$y_t^W$	$\tilde{i}_{t+1}^*$
$y_t^W$			.397 (.187)
$\tilde{\pi}_t^{CPI*}$		-.446 (.537)	1.090 (.592)
$\tilde{\pi}_{t-1}^{CPI*}$	.370 (.196)	.244 (.544)	.063 (.484)
$y_{t-1}^W$	.047 (.134)	1.386 (.240)	-.057 (.340)
$\tilde{i}_t^*$	.100 (.082)	-.037 (.283)	.511 (.313)
$\tilde{\pi}_{t-2}^{CPI*}$	-.022 (.206)	.186 (.362)	.971 (.473)
$y_{t-2}^W$	.030 (.211)	-.243 (.300)	.028 (.393)
$\tilde{i}_{t-1}^*$	-.106 (.114)	-.293 (.399)	.090 (.472)
$\tilde{\pi}_{t-3}^{CPI*}$	.373 (.176)	.111 (.325)	.266 (.563)
$y_{t-3}^W$	-.053 (.133)	-.180 (.232)	-.337 (.273)
$\tilde{i}_{t-2}^*$	.018 (.101)	.274 (.212)	.108 (.223)

Figure 1. Markup and hours

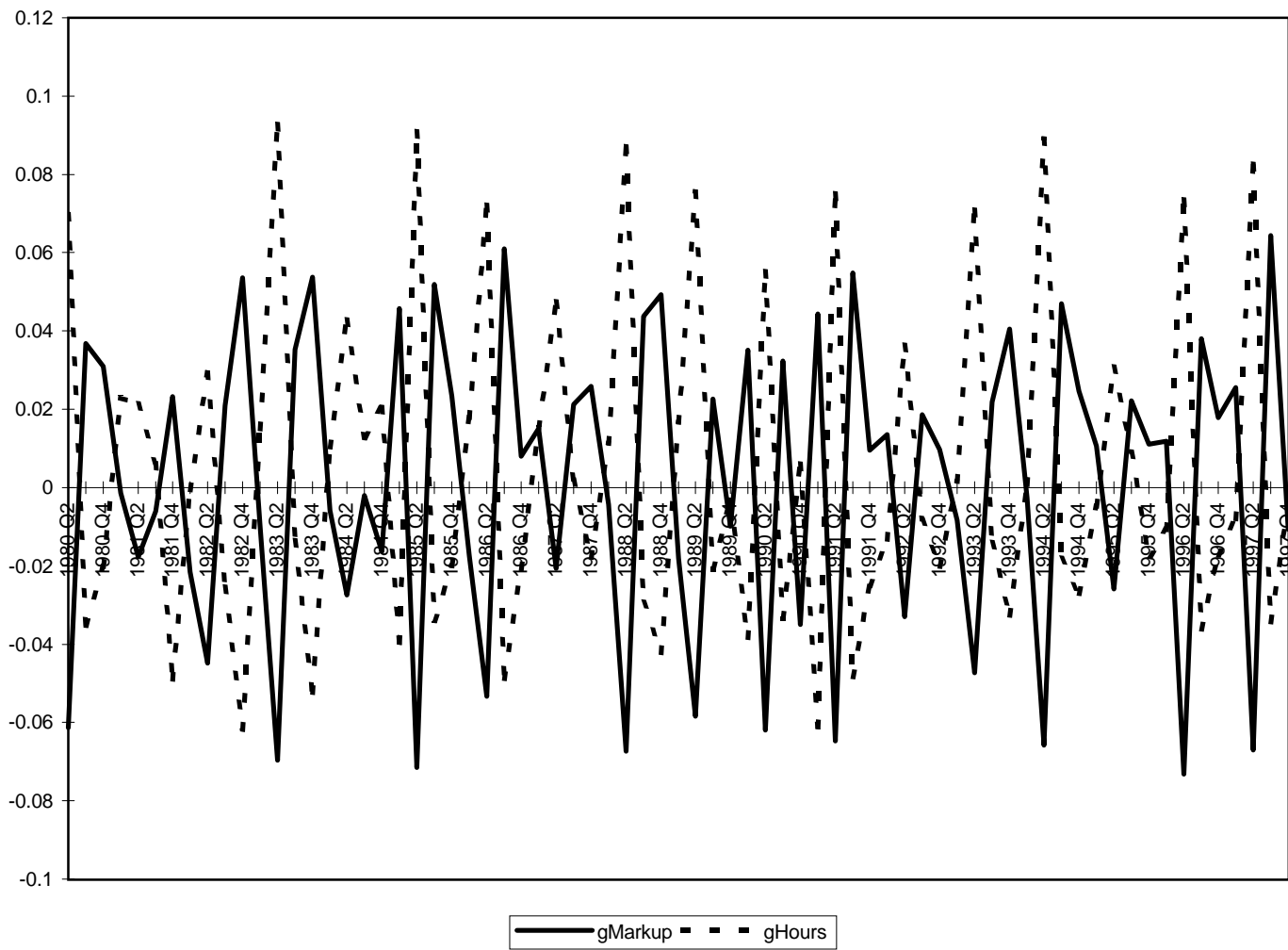


Figure 2. OMEGA

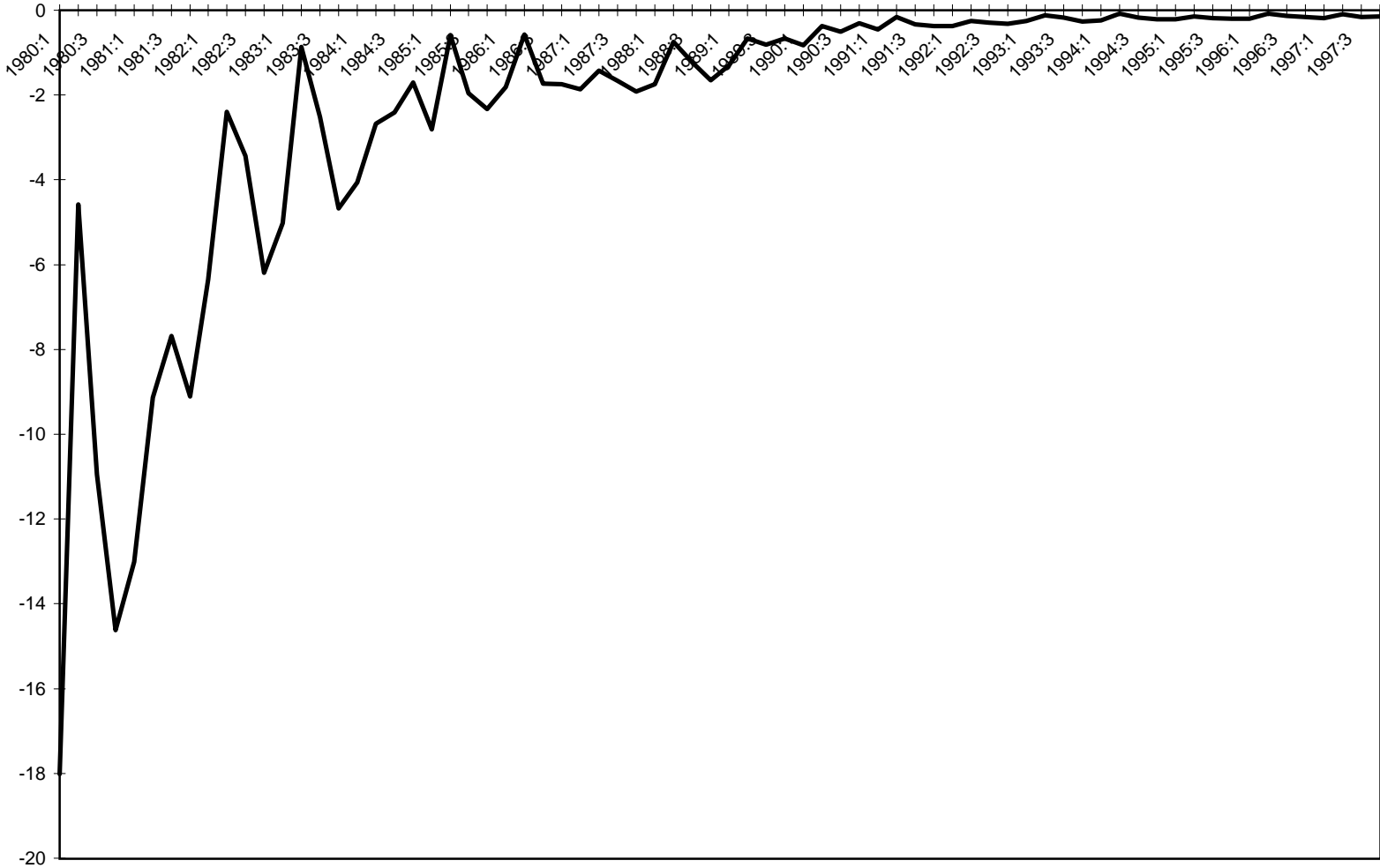


Figure 3.a. Investment and average  $q$

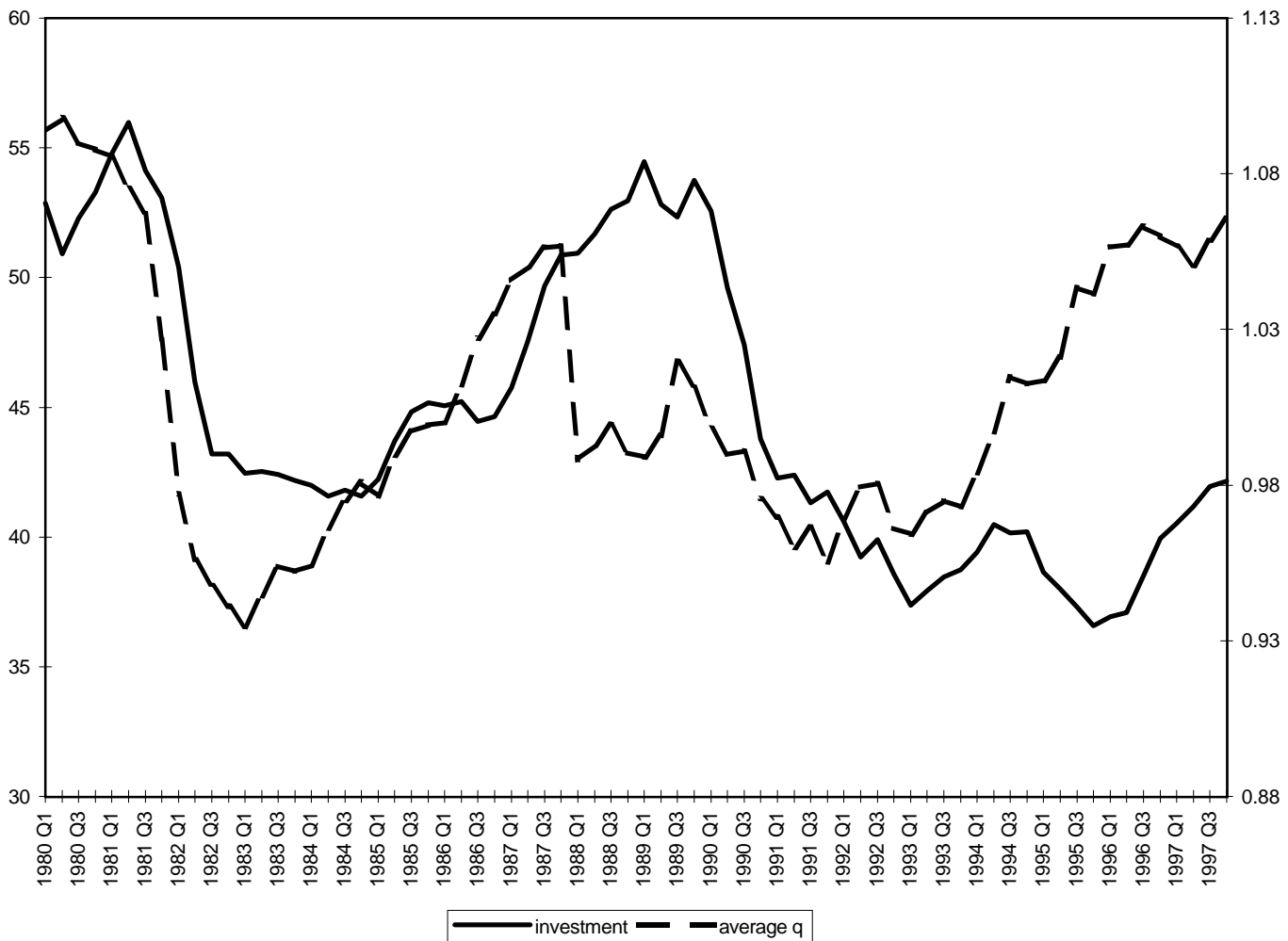
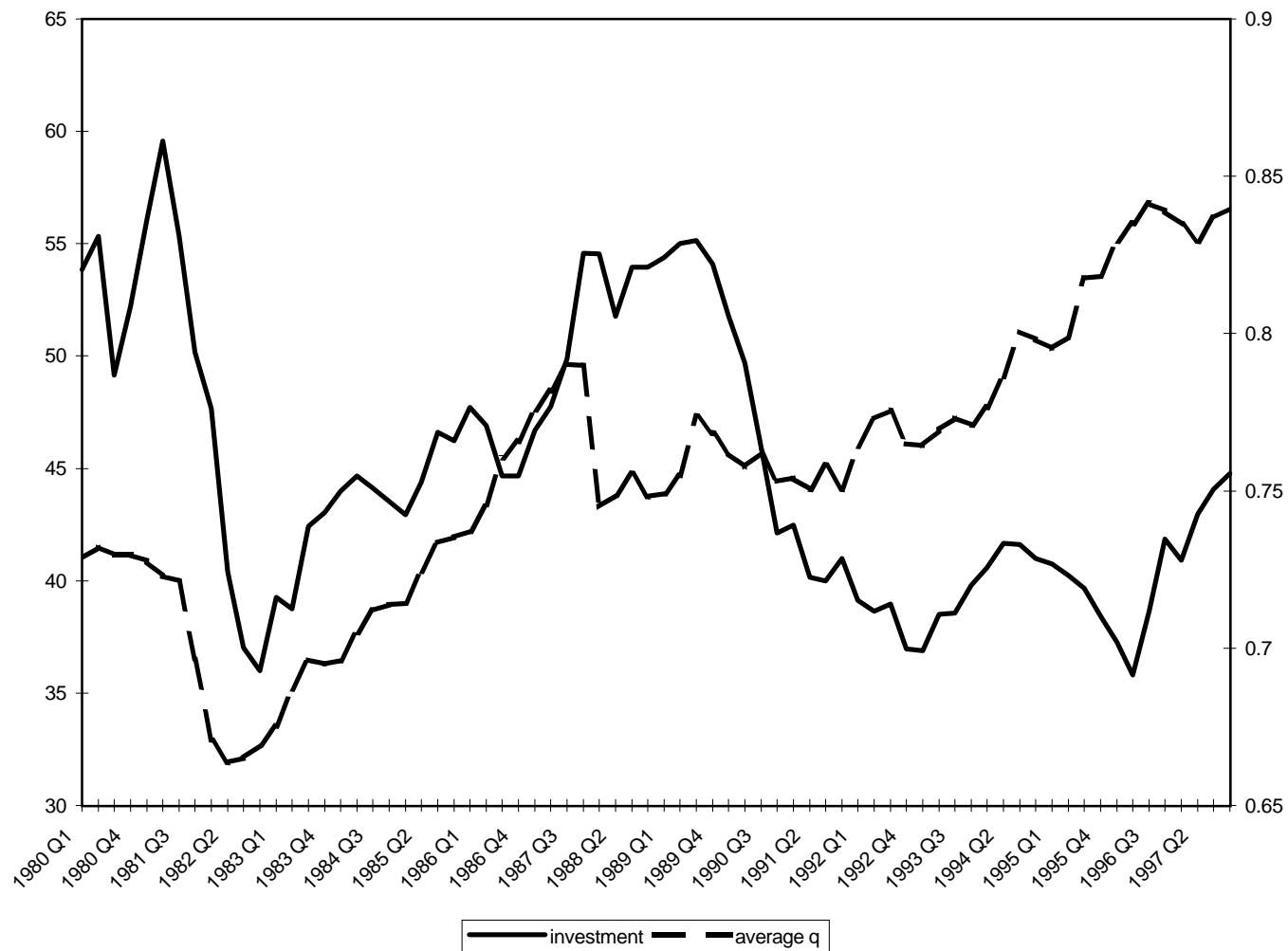
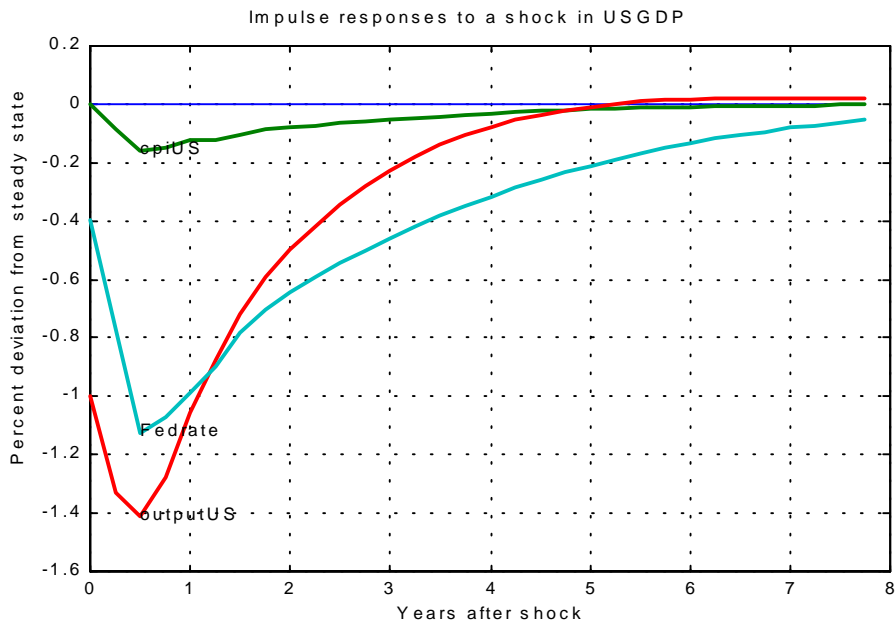


Figure 3.b. Investment and average  $q$ , adjusted for inventories



**Figure 4.** Impulse responses, U.S. economy, shock to U.S. GDP



**Figure 5. Impulse responses, strict inflation targeting, shock to U.S. GDP**

