Economics Department

Working Papers in Economics

Boston College *Year* 1996

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revised 10/6/1996

Abstract

We employ threshold cointegration methodology to model the policy problem solved by the Federal Reserve System in their manipulation of the discount rate under a reserves target operating procedure utilized since October 1979. The infrequent and discrete adjustments that characterize movements in the discount rate instrument vis-a-vis the Federal Funds rate do not lend themselves to a linear cointegration framework. The inherently nonlinear relationship arising from the Fed's self-imposed constraints on discontinuously changing the discount rate is satisfactorily modelled as an instance of threshold cointegration between the discount rate and the Federal Funds rate.

[§] The order of the authors' names is alphabetical. We thank Bruce Hansen, John T. Barkoulas, David A. Belsley, participants in the Second International Conference on Computing in Economics and Finance and participants in the Eleventh Congress of the European Economic Association for their comments; the standard disclaimer applies. Please address correspondence to Baum at Department of Economics, Boston College, Chestnut Hill MA 02167 USA, tel. +1 617 552 3673, fax +1 617 552 2308, email baum@bc.edu.

Modelling Federal Reserve Discount Policy

1. Introduction

During the past three decades, the Federal Reserve has altered its reliance on alternative tools of monetary policy on several occasions. The efficacy of the discount rate, the Fed's original policy tool, has varied throughout monetary regimes, but has remained an important instrument in the current policy environment. Changes in the discount rate influence market participants' expectations formation, and, under some circumstances, directly affect commercial banks' behavior. Although an extensive literature on the interactions of monetary policy instruments has developed, the particular characteristics of discount policy warrant analysis with current econometric methodologies.

In this paper, we explicitly take account of the infrequent nature of Federal Reserve discount policy actions and the asymmetric effects that those actions have on the market for bank reserves. A band threshold autoregressive (Tong, 1990) model is used to capture these characteristics, and the technique of threshold cointegration (Balke and Fomby, 1994) is used to model the joint evolution of the discount and Federal funds rates in the post-October 1979 policy regimes.

The post-October 1979 period, which has been characterized by reserve targeting behavior of the Fed, implies a cointegrating relationship between the discount and Federal funds rates. However, standard linear cointegration techniques are not designed to capture the discrete adjustments of the discount rate. The Band-TAR threshold cointegration model captures these discrete adjustments by modelling the equilibrium error of the cointegrating relationship. The long-run relationship between these two rates is non-responsive to adjustments either in the discount rate or the instruments directly affecting the Federal funds rate for observations within some band. However, once the deviation from the long-run equilibrium of the Federal funds – discount rate spread exceeds the limits of this band, the cointegrating relation restores the spread to lie within the band. The band limits or thresholds in the Band-TAR model, specified as lagged values of the equilibrium error, are estimated as part of the model. The estimated lag parameter indicates the lag in the reaction of the Fed to deviations of the discount and Federal funds rates from the equilibrium relationship.

The plan of the paper is as follows. Section 2 reviews the relevant literature and presents considerations which policymakers must take into account under alternative operating regimes in the context of an analytical model of the effects of the discount rate on the market for reserves. In Section 3, we review the threshold autoregressive (TAR) model and its Band-TAR variant, and present its extension to threshold cointegration. Section 4 presents our implementation of the model on weekly U.S. data for the 1979-1995 period. Conclusions are presented in Section 5.

2. Review of the literature

The discount rate (the rate charged on banks' borrowed reserves) is the Federal Reserve's original instrument of monetary policy, but has been largely ignored relative to open-market operations in textbook discussions of policymaking in the postwar era. Although the discount rate's role is often described as purely an "announcement effect," a well-publicized signal to the markets, there is a sizable body of literature within the Fed that considers the discount rate of greater importance than this. In the past 25 years, as the Fed has varied its operating policies, policymakers have often followed a strategy giving the discount rate an explicit role in the money supply process. We now survey elements of this literature.

The defining event in any analysis of recent U.S. monetary policy was the October 6, 1979 shift in Federal Reserve operating policies, taken in response to rapid money growth, accelerating inflation, and a weakening dollar (Cook, 1989, p. 3). The new procedures led to an abandonment of explicit targets for the Federal funds rate and a new emphasis on the management of bank reserves: specifically, the volume of nonborrowed reserves (NBR). This shift led to marked increases in the level and volatility of interest rates at all tenors, volatile growth rates for the several monetary aggregates, and over time to a sharp drop in the inflation rate. It is generally accepted that the Fed's procedures again shifted on October 9, 1982, when the behavior of the narrow money stock (M1) was further deemphasized and the operating target was modified to borrowed reserves (BR) (Peristiani, 1991, p. 13). Since that latter shift, the broad outlines of monetary policy have been unchanged, although pronouncements of Fed officials suggest that multiple policy objectives are constantly being weighed in the policymaking process.

Under the post-October 1979 regimes (either NBR or BR), monetary policy has operated via quantity adjustments in reserve measures, with money market rates varying to equilibrate the market for reserves. This process has been described as "indirect funds rate targeting," in which the Fed "...estimates the banking system's demand for reserves and provides the bulk of those reserves through open market purchases. But it forces the banking system to borrow a small fraction from the discount window...for a given discount rate, targeting borrowed reserves allows the Fed to target the Federal funds rate indirectly." (Goodfriend, 1991, p.19) However, under this regime (as contrasted with direct funds rate targeting) "...it is not as obvious to the market what the target is." (Goodfriend, op.cit.) Although this argument applies directly to the borrowed reserve regime, there is evidence that the 1979-1982 nonborrowed reserve regime was, in practice, largely a borrowed reserve targeting scheme; Cook (1989, p.4) found that "while some of the movement in the funds rate over this period resulted from the automatic adjustments, most of the movement–roughly two-thirds–was due to judgmental actions of the Federal Reserve." Thus, although there might be regime-shift effects during the post-October 1979 period, it appears reasonable to treat the entire period as an episode in which the volume of borrowed reserves plays a role in determining money market interest rates.

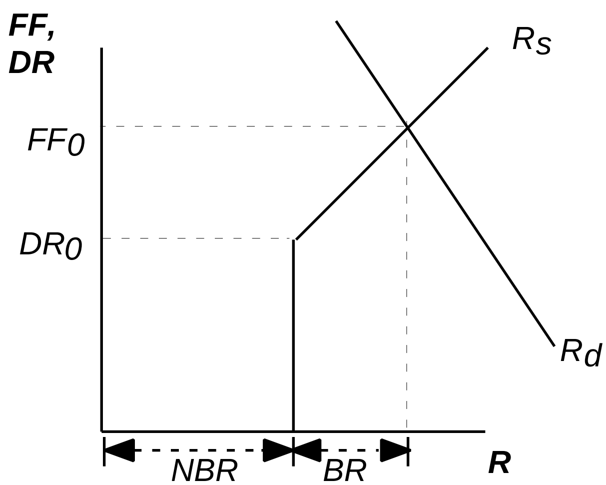
Monetary policy actions directly affect the aggregate stock of bank reserves, which in turn influences the levels of the monetary aggregates and the quantity of lending. Under a reserves targeting procedure, shifts in reserve demand (e.g., changes in the public's demand for narrow money) directly influence the Federal funds rate and, indirectly, other money market interest rates. Although the Fed could choose to control total reserves (or the monetary base), policymakers have used procedures which smooth funds rate movements – ranging from the 1970s' funds rate targeting procedure that allowed rates to vary only in a narrow band to more recent strategies in which nonborrowed or borrowed reserves were targeted.[1]

Total reserves supplied to the banking system are comprised of nonborrowed reserves (NBR) plus those borrowed at the discount window (BR). The Federal funds market allows reserves to be shifted among banks, but does not alter the total quantity of reserves. The demand for reserves is taken to be an inverse function of

[1] As Goodfriend and Whelpley (1986) point out, earlier procedures such as "free reserve targeting" used by the Fed in the 1920s, 1950s and 1960s were analytically similar to the current borrowed reserve targeting procedures.

the Federal funds rate, whereas the provision of reserves involves two components: those supplied inelastically by the Fed (NBR) plus those borrowed at the discount window (BR), with the latter considered interest-sensitive. Figure 1 (Goodfriend and Whelpley, 1986, p.6) illustrates this relation between reserves (R), the discount rate, and the funds rate.

Figure 1



Under a nonborrowed reserve operating target, such as that used in the October 1979-1982 period, the Board staff formulated a forecast of expected borrowing (derived in part from recent discount window experience), and established a "...nonborrowed reserves path which (when added to this expected level of borrowing) will provide the total reserves thought to be consistent with the [Federal Open Market] Committee's money growth targets." (Keir, 1981, p. 2) As long as the supply of NBR falls short of the demand for total reserves at the current discount rate, borrowing will be positive, and the funds rate will exceed the discount rate. The supply curve for reserves will be neither vertical (as it would be if total reserves were targeted) nor horizontal (as it would be under a funds rate target), but upward-sloping as shown in the Figure, reflecting the response of the funds rate to a higher quantity demanded of interbank borrowing. As Goodfriend and Whelpley point out (1986, p.6), nonborrowed reserve targeting is a cross between funds rate and total reserve target procedures.

Since late 1982, the predominant operating procedure has been borrowed reserve targeting, in which the provision of nonborrowed reserves is altered (via open market purchases or sales) to keep aggregate discount window borrowing unchanged in the face of reserve demand shifts. For a given level of the discount rate and positive demand for borrowed reserves, the funds rate is determined by a borrowed reserve target, changing only in response to variations in the demand for reserves (or variations in the Fed's tolerance for individual banks' borrowing patterns). As long as nonborrowed reserves fail to satisfy banks' demand for reserves, the funds rate will lie above the discount rate, and changes in the discount rate will have a direct effect on the funds rate. Discount rate adjustments will have a direct effect on the funds rate in either a NBR or BR operating regime (although the magnitudes of the funds rate response may differ between regimes).

In contrast, when nonborrowed reserves satisfy banks' demand for reserves, the funds rate will fall below the discount rate, and adjustments of the discount rate will have no direct effect on the funds rate; banks would have no incentive to borrow at the discount window. Under this scenario, discount rate changes would have their usual announcement effect, but even decreases would not trigger borrowing unless the new rate was sufficiently low to restore the situation pictured in Figure 1. This asymmetry implies that the spread between the funds rate and the discount rate, $(FF - DR)$, will affect the provision of reserves only when it is positive. The spread was found to have significant explanatory power for member bank borrowing during the interest rate targeting regimes, with more than 80 per cent of borrowing explained by the spread alone over the 1970 – September 1979 period (Keir, 1981, p.A-1). In the subsequent NBR and BR targeting regime, the equation should be inverted, reflecting the endogeneity of the funds rate under those regimes. Board researchers found that the relationship between borrowing and money market spreads became more variable, consistent with the increased volatility of interest rates experienced in the post-October 1979 period.

In either NBR or BR regimes, a positive $(FF - DR)$ spread is proportional to the benefit of a net saving on the interest cost of reserves received by patrons of the discount window. In practice, the Fed has discouraged frequent and sizable borrowings, either by threatening greater surveillance of banks' financial status or through an explicit penalty surcharge (as used in the early 1980s). Researchers have described member banks' borrowing behavior in terms of a dynamic optimization problem (cf. Goodfriend, 1983), where desired borrowing depends upon recent borrowing history and projected reserve needs. Here banks are sensitive to both explicit restrictions on borrowing (such as penalty surcharges) and implicit discouragement of borrowing through the credible threat of heightened surveillance.

We now discuss the Federal Reserve's discount policy problem, focusing on the recent history of the instrument as a policy tool. We do not consider normative elements here, but rather the manner in which the instrument has actually been used. The policy problem implicitly solved in this process is quite interesting, since discount rate adjustments are unusual events by design: they are widely spaced, discrete in magnitude, and generally do not correct for earlier adjustments (i.e., the sequence of discount rate changes has few sign changes). Policymakers are said to dislike "whipsawing the market," so that "A target change establishes the presumption that, absent significant new information, the target will not be soon reversed." (Goodfriend, 1991, p. 10) Some descriptive measures of discount rate changes in the October 1979 – January 1996 period are presented in Figure 2 and Tables 1 and 2, while Figure 3 presents the $(FF - DR)$ spread over that period. There are 40 changes among the 842 weeks of the period, the most frequent being a 50 basis point cut. Discount rate settings are most commonly in force for one to two months, as Table 2 indicates: 45 per cent of the no-change periods were no more than eight weeks' duration.

Figure 2
Discount Rate, Oct. 1979 – Jan. 1996

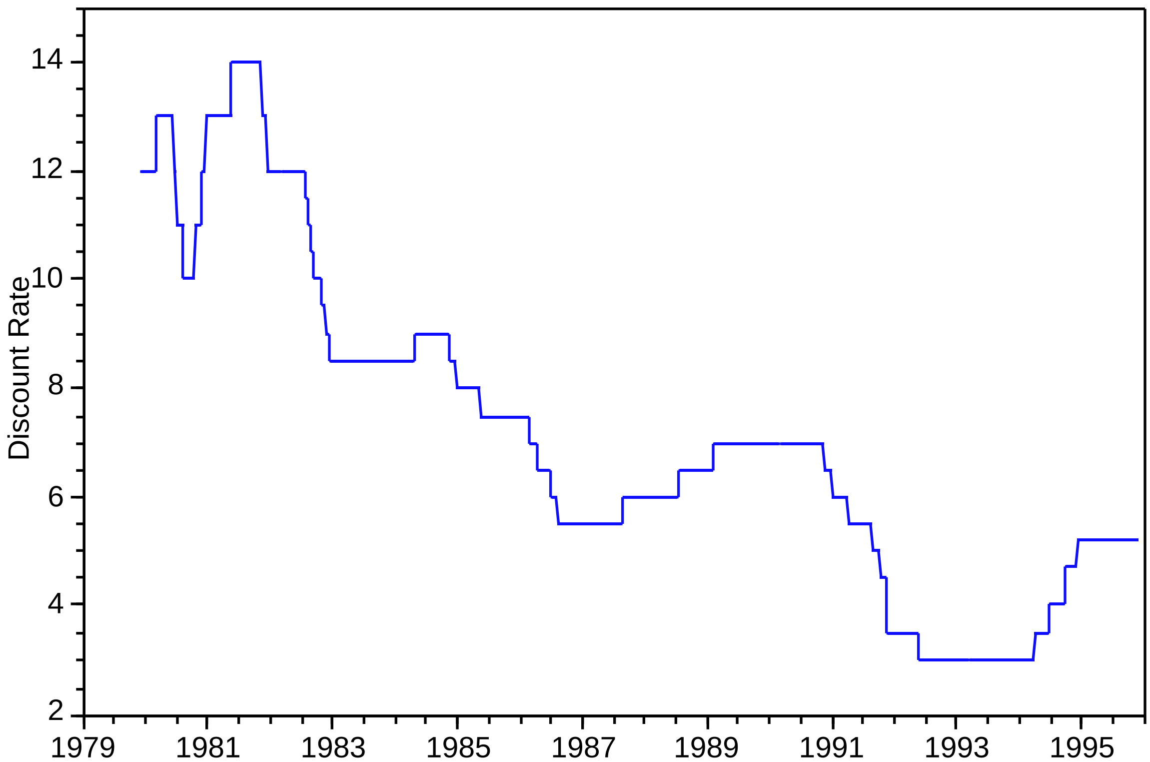


Figure 3

Fed Funds – Discount Rate Spread, Oct. 1979 – Jan. 1996

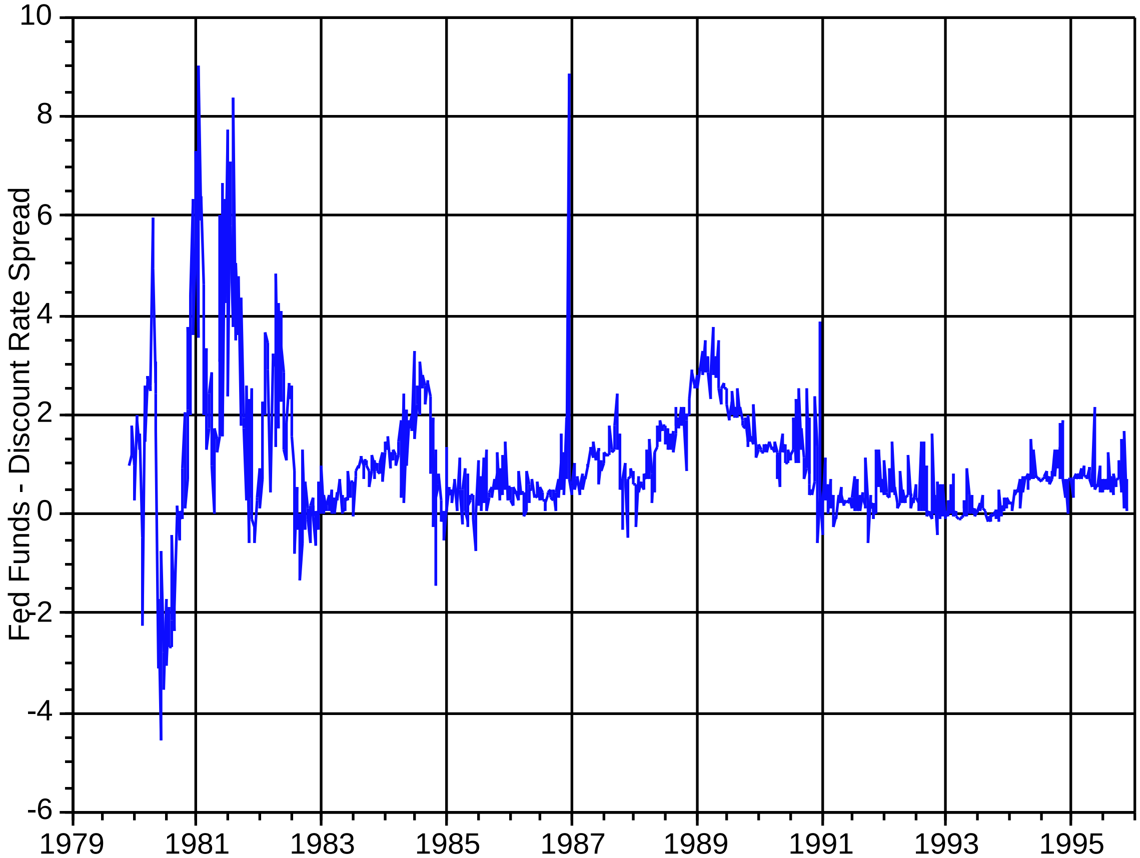


Table 1
Federal Reserve Discount Rate Changes, 10/3/1979 – 1/17/1996

Weekly Change in Discount Rate, bp	Frequency (weeks)	Per cent
-100	6	0.71
-50	20	2.38
0	802	95.25
+50	7	0.83
+75	1	0.12
+100	6	0.71
Total	842	100.00

Notes: data taken from Federal Reserve H.15 release.

Table 2
Duration of Discount Rate Settings, 10/3/1979 – 1/17/1996

Number of Weeks of Unchanged Rate	Frequency (weeks)	Per cent
2-4	8	20.0
5-8	10	25.0
9-12	4	10.0
13-16	3	7.5
17-20	2	5.0
21-24	2	5.0
25-28	2	5.0
29-32	3	7.5
> 32	6	15.0
Total	40	100.0

Notes: Derived from H.15 data. There were no instances of rate changes in successive weeks. The maximum number of weeks without a rate change was 96 (7/22/92 – 5/18/94).

These descriptive characteristics of discount rate policy may be better understood by considering the context in which these changes take place. More frequent adjustments in the discount rate would clearly diminish the announcement effect of a change. In the current environment, such rate changes are news items because of their infrequent and discrete nature. The Fed's unwillingness to carry out what might appear to be contradictory policy actions – a rate increase followed by a smaller decrease, for example – only strengthens the notion that the observed discount rate changes are almost by design "too little, too late." An observed policy instrument with these characteristics provides the applied econometrician with a latent variable problem; that is, we may distinguish between the discount rate needed to carry out policy, based on the current instrument set, d^*, and the actual discount rate, d. The actual rate will be adjusted toward d^* only when the gap is sizable enough to trigger a policy response, and only a portion of the gap is likely to be closed in any single adjustment.[2] A further complication arises because of the asymmetric effects of the discount rate: Fed policymakers may be more cautious in altering the discount rate when that rate change has a near one-to-one effect on the funds rate than they would when only an announcement effect is involved.

In summary, the existing monetary policy literature establishes that the discount rate as a policy instrument will have differential effects on money market rates and the market for reserves depending on its relation to the funds rate. An evaluation of discount rate policy in the period since October 1979 illustrates that discount rate changes have been relatively infrequent, and that their magnitude is consistent with a process of partial adjustment, placing a sizable cost on overshooting and policy reversals.

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[2] In a monthly money market model fit over the 1975-1980 period, Tinsley et al. used a partial adjustment framework to summarize the historical policy rule for the discount rate, and found that the monthly speed of adjustment of DR towards the current funds rate implied a mean lag of about 3.5 months. (1982, pp. 843-844)

3. Threshold cointegration of money market rates

Under reserve targeting strategies, the relationship between the discount rate and the funds rate entails their comovements – either through adjustment of the funds rate in the market for Federal funds or the Fed's altering the discount rate in line with market forces and their target path for nonborrowed or borrowed reserves. If the discount rate and funds rate exhibit nonstationarity, those comovements would imply cointegrating behavior. In this section, we present evidence of such nonstationarity, and discuss a threshold cointegration framework used to model comovements of the rates. Although the funds rate is free to continuously adjust to conditions in the market for reserves, the discount rate is determined within a policy regime that implicitly places a high cost on frequent and / or small adjustments of the rate. Therefore, discount rate behavior vis-à-vis that of the funds rate must be modelled in a framework that allows for infrequent and discrete adjustments of the discount rate. To proceed with cointegration analysis, we must establish that both discount and funds rates are integrated of the same order.

3.1. Stationarity properties of the discount and funds rates

Table 3 presents the results of a battery of tests on the discount rate and Federal funds rate for the period October 1979 – January 1996 and over two subperiods, October 1979 – October 1982 and November 1982 – January 1996, that bracket the break between he nonborrowed reserve and borrowed reserve operating policies. The tests on the discount rate almost uniformly support the hypothesis of a unit root process. The inability to reject the unit root null for the ADF and PP tests, combined with the general rejection of the KPSS test's null of stationarity leads to a clear conclusion of I(1). The tests are somewhat contradictory for the funds rate. The PP tests generally support stationarity, while most of the ADF tests and all but one of the KPSS tests indicate a unit root. To further explore the funds rate's time series properties, we apply the test of Geweke and Porter-Hudak (1983) for fractional integration. This test, executed with power={0.50,0.55,0.60} on the differences of the series, almost never rejects the null of a single unit root, and thus supports the hypothesis of an integer order of integration against the alternative of fractional integration. We conclude that both the discount rate and funds rate may be modelled as unit root processes over the sample period.

3.2. The threshold cointegration model

Although the preceding evidence supports modelling the discount and funds rates as integrated processes, our understanding of the policy process in which the discount rate is set calls into question applying standard linear cointegration techniques. In the standard Engle-Granger (1987) framework, a disequilibrium in the cointegrating relationship triggers adjustment via the error correction process, and this continuous process (even if sampled in discrete time) tends to restore the equilibrium along a smooth trajectory. By contrast, when there are fixed costs of adjustment (including policymakers' reluctance to initiate adjustments) a disequilibrium can persist until a threshold is reached. This logic applied to nonstationary processes is termed threshold cointegration (TCI), and the equilbrium error that produces adjustment in the standard cointegrating framework "follows a threshold autoregression that is mean reverting outside a given range and a unit root inside this range." (Balke and Fomby (1994), p.1) In this section, we sketch the econometric background for this framework along with the methodology used for detecting the nonlinearities of the threshold autoregression (TAR) model and for identifying its appropriate order.

In its simplest form, threshold cointegration (cf. Balke and Fomby, p.4) can be modelled as a variant of an Engle-Granger bivariate system for (y_t, x_t),

$$\begin{aligned} y_t + \alpha x_t &= z_t, \text{ where } z_t = \rho^{(i)} z_{t-1} + \varepsilon_t, \\ y_t + \beta x_t &= B_t, \text{ where } B_t = B_{t-1} + \eta_t, \end{aligned} \tag{1}$$

where the deviation from the equilbrium relationship between y and x is given by z, the common trend of the system is B_t, and the cointegrating vector is given by $(1,\alpha)$. The presence of threshold cointegration is derived from the auxiliary relation

$$\rho^{(i)} = \begin{cases} 1 & \text{if } |z_{t-1}| \leq \theta \\ \rho & \text{if } |z_{t-1}| > \theta \end{cases} \quad \text{where } |\rho| < 1. \tag{2}$$

Departures from equilbrium follow a random walk if z falls short of the threshold value (in absolute value terms) but follow a mean reverting process beyond the threshold. For large values of the equilibrium error, y and x are cointegrated. In this case, the threshold values are symmetric around zero, the coefficients in the mean-reverting law of motion are identical for positive and negative threshold values, and a single threshold is present; all of these constraints may be relaxed in a more general framework.

As Balke and Fomby suggest, threshold cointegration may appear in many circumstances where private agents or policymakers face explicit or implicit costs of action. Dynamic control problems in which there are fixed or linear costs of adjustment (e.g. S,s models) may give rise to such behavior on the part of consumers, unions, or firms. Likewise, there are many instances where the costs of policy actions (e.g., the political capital that must be expended to alter the tax system) may hinder appropriate adjustment until a relatively high threshold has been reached. In financial markets, examples of exchange rate management via target zone models (Karasulu, 1995), price stabilization, and monetary policy (specifically, the discount rate / funds rate relationship) are all cited by Balke and Fomby as plausible examples of TCI. The cointegrating relationship in such a framework is dormant within a certain range of small disequilibria, but is activated when the system crosses the threshold. Such actions may be asymmetric, in the sense that thresholds may not be evenly spaced; for instance, a financial firm might take quick action to replenish its working capital if it fell below a certain value, but might allow it to build to a much higher level before redeploying its assets.

In the context of monetary policy, the model illustrated above (which Balke and Fomby (1994, p.8) term the "equilibrium TAR", or EQTAR) might not be the most reasonable framework. In EQTAR, the equilibrium error process is driven back toward a single target value when it breaches the threshold. Here it might be more appropriate to consider a variant on this approach, the "Band-TAR," in which the equilibrium error process is driven back toward a target band: that is, the target is a range of values, and the policymaker only tries to drive the relationship back within that range. Given the uncertainty in the relationship between the magnitude of the $(FF - DR)$ spread and the quantity of borrowing, such a description of monetary policymakers' actions makes sense. For a symmetric band, the Band-TAR model can be written as:

$$z_t = \begin{cases} \theta(1-\rho) + \rho z_{t-1} + \varepsilon_t, & \text{if } z_{t-1} > \theta \\ z_{t-1} + \varepsilon_t, & \text{if } |z_{t-1}| \le \theta \\ -\theta(1-\rho) + \rho z_{t-1} + \varepsilon_t, & \text{if } z_{t-1} < -\theta, \end{cases} \tag{3}$$

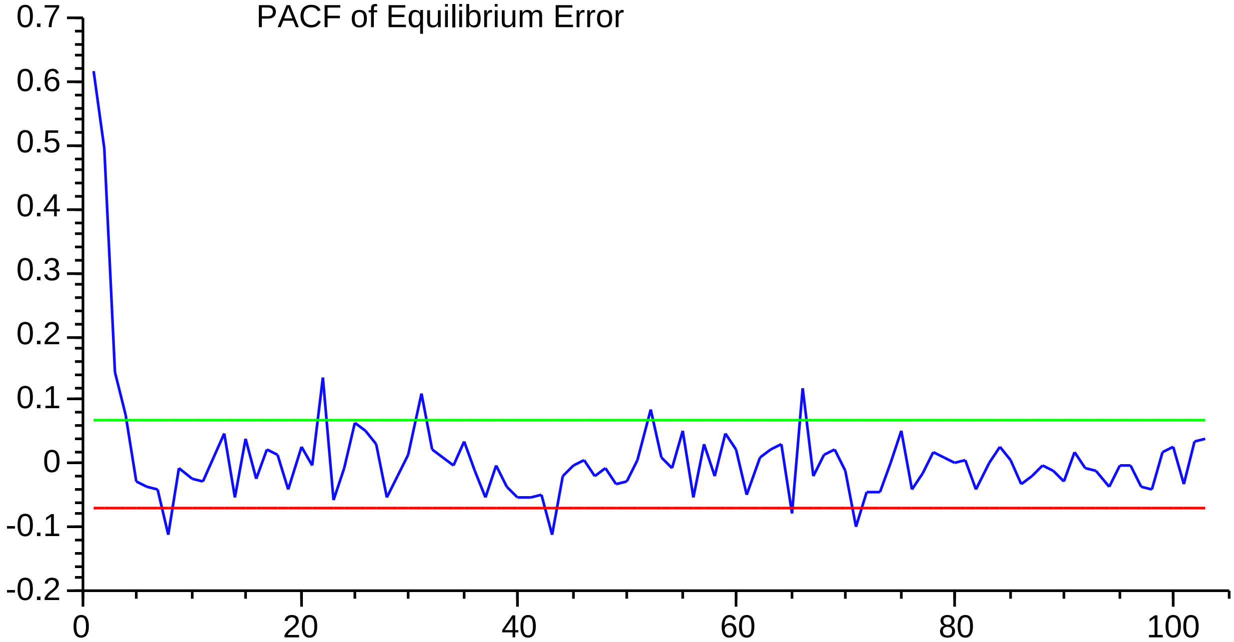
where the band is the interval $\{-\theta, \theta\}$. Balke and Fomby show that although both the EQTAR and Band-TAR models yield a stationary equilibrium error, the Band-TAR entails a very different dynamic relationship for z, with considerably greater persistence than its EQTAR counterpart.

In modelling the dynamic relationship between the discount rate and funds rate, we might feel that the symmetry of this Band-TAR is too restrictive. The asymmetric effects of discount rate changes on the funds rate calls for an asymmetric band. So rather than a single bandwidth parameter θ, we estimate separate positive and negative thresholds from the data.

3.3 Identification of the Band-TAR model

Before applying threshold cointegration techniques to our sample, we must determine whether appropriate nonlinear behavior is present in the data. Within the sizable literature on threshold autoregressive (TAR) models and their variants (e.g. Tong (1983), Priestley (1988), Tsay (1989)), a testing strategy has arisen that examines the relationship between the variable of interest and the lagged value of a threshold variable, which may itself be the variable of interest, in the case of a "self-exciting" TAR, or SETAR. In our model, both the variable of interest and the threshold variable are the equilibrium error series, defined as the least squares residual from the regression of the funds rate on the discount rate. An autoregression is run on the equilbrium error process, with the order (p) selected from examination of its partial autocorrelation function (PACF) and the Akaike information critierion (AIC) for varying lag lengths. The PACF of the equilibrium error series, estimated for 104 lags, is shown in Figure 4, with its asymptotic two standard error bands. Although there are significant partial autocorrelations throughout the sequence (repeating on a roughly 13-period delay) the initial extremum at the eighth lag appears to be indicative of the equilibrium relationship's behavior. This matches the casual evidence given in Table 2, in which a modal frequency of realignment in the eight week range is apparent. Although the PACF evidence does not clearly indicate the appropriate order of autoregression, it is indicative of an important frequency in the underlying relationship.

Figure 4
PACF of Equilibrium Error



To tie down the AR order, we use the AIC measure, fit to up to 40 lags of the equilbrium error in an AR(*p*) model. The minimum AIC occurs for a *p* equal to 31 weeks. A secondary minimum is apparent at 23 weeks, and a third at 11 weeks. The AIC values change very little over the entire range; the values for p=23 and p=11 exceed the minimum by only 2.8 per cent and 4.6 per cent, respectively. Thus, to help keep the ensuing analysis of results manageable, we take 11 weeks as the appropriate order for the preliminary autoregressive model. This specification serves as the baseline for Tsay's (1989) methodology of fitting the TAR: conditional on *p*, we then search for an appropriate value of the delay parameter, *d*.

We estimate "arranged autoregressions" for the equilibrium error series by reordering the sample according to the threshold variable for a given delay parameter *d*, and generate a sequence of recursive least squares estimates via a simple application of the Kalman filter. These estimates give rise to a sequence of standardized recursive residuals which are then regressed on *p* lags of the original series. The resulting sequence of ANOVA F statistics is then used to establish the existence of threshold nonlinearity, and then to determine the appropriate setting of the delay parameter, *d* (Tsay, 1989, p.235). Conditional on a *p* of 11 weeks, we find a maximal F-statistic of 3.517 associated with a delay parameter (*d*) of seven weeks – a value broadly consistent with the casual evidence in Table 2 of the duration of non-change episodes.

Having specified the order of the threshold autoregression *p* and the delay parameter *d*, the model can be used to determine the number and location of the threshold values. The tests just described are consistent with one or more threshold values: that is, two or more regimes. In our case, we would expect to find two threshold values, corresponding to the boundaries of the cointegrating relationship that persists only for extreme values of the equilibrium error. These thresholds are identified from study of scatterplots of various statistics against the threshold variable: the standardized predictive residuals and the "t" ratios of recursive estimates of an AR coefficient. As Tsay (1989) discusses, the predictive residuals are biased at the threshold values, making it possible to spot the thresholds from their scatterplot. Likewise, the recursive "t" ratio tends to jump in the vicinity of a threshold value, while it should gradually converge on the full-sample value within a regime.

Conditional on p=11 and d=7, we find evidence of two threshold values from the recursive "t" ratios for those AR parameters that are clearly significant throughout the sample. Other evidence from Tsay's technique allows us to pick specific values for these breakpoints. Six upper and four lower thresholds are picked as shown in Table 4. Summary statistics in the table show the number of weeks in which the equilibrium error breached the threshold along with the mean and median values of the $(FF - DR)$ spread during those excursions. From these threshold values, it appears that the upper threshold – which tends to result in a discount policy response when breached – requires that the funds rate rise at least 210-230 basis points above the discount rate, yielding a strong incentive for banks to turn to the discount window to avoid the high costs of interbank borrowing. In contrast, the lower threshold appears to apply to a rather modest decline in the funds rate, keeping the spread within roughly 20 basis points and so robbing the discount rate of any direct effect on the market for reserves.

Now, we turn to estimating the Band-TAR model based on this specification of the process governing the evolution of the discount rate and funds rate.

4. Estimation of the Band-TAR model for discount policy

Since our methodology to identify the lag structure, the delay parameter, and the threshold values does not yield unique estimates of the threshold values, we search over the upper and lower threshold values listed in Table 4, estimating a set of models for each regime (lower, middle, upper) for each combination of the lower and upper threshold values – yielding 24 distinct models, each containing three segments of the threshold autoregression. As Tong (1983) and others have noted, there is no need to restrict the orders of the autoregressive models for each discrete segment to be identical. Thus, for each of the three segments, we search over the lag order, selecting the model that minimizes the Akaike criterion. A composite score for each model is then defined as the sum of its AIC scores over its three segments. The 24 models are then ranked according to their composite scores. The minimum composite score was attained by a model with an upper threshold of 0.3435 and a lower threshold of –0.2587. The model can be expressed as:

$$z_t = \begin{cases} \alpha_l + \sum_{j=1}^{3} \beta_{lj} z_{t-j} + \varepsilon_{lt}, & \text{if } z_{t-7} \leq -0.2587 \\ \alpha_m + \sum_{j=1}^{2} \beta_{mj} z_{t-j} + \varepsilon_{mt}, & \text{if } -0.2587 < z_{t-7} \leq 0.3435 \\ \alpha_u + \sum_{j=1}^{2} \beta_{uj} z_{t-j} + \varepsilon_{ut}, & \text{if } z_{t-7} > 0.3435 \end{cases} \quad (4)$$

Summary statistics for this model are presented in Table 5. The thresholds divide the sample into three subsamples of similar size; the portions of the model for the lower and upper threshold regimes fit considerably better than that for the middle regime in terms of R^2, but the same is not reflected in the magnitude of the standard error of estimate. The Ljung-Box Q statistic indicates no evidence of autocorrelation in any of the subsamples. The composite Akaike criterion value of 3936.66 that leads to this model's selection reflects its performance over all three regimes. There are other combinations of threshold values and AR lags that produce greater predictive accuracy for a particular regime, but none tested improves upon this model's overall predictive ability.

We can now turn to estimating the vector error correction model that expresses the equilibrating relationship between the Federal funds rate and discount rate. This model is specified, for each of the three regimes, as a VAR in $\{\Delta FF, \Delta DR\}$ with the lagged equilibrium error as a deterministic variable. The lag order of the VAR in each regime is selected by a likelihood ratio test at the 10% level of significance, resulting in three, two, and one lags in the lower, middle, and upper threshold regimes, respectively. Results from estimating this set of VECMs are presented in Table 6.

The model fits satisfactorily in all three regimes for both of the dependent variables, although, as expected, the explanation of the Fed funds rate is the more successful, given the infrequency of changes in the discount rate. This may account for the insignificance of lagged discount rate changes in the discount rate equation, whereas those changes are consistently influential in the Fed funds rate equation. The model is considerably more successful in explaining the post-October 1982 borrowed reserves regime than the turbulent 1979-1982 period in which nonborrowed reserves targeting was in place and the Federal funds rate exhibited considerable volatility. Although the theoretical justification of the Band-TAR specification suggests that the lagged equilibrium error would be insignificant in the middle regime, the data reject that constraint for both series. We take this as evidence of nonlinearity in the bivariate relationship, perhaps because the error correction coefficient takes on different values between the lower, middle, and upper regimes – even if the coefficients from the middle regime are nonzero. Thus this Band-TAR based model of the cointegrating relationship between the discount rate and Federal funds rate seems able to capture the inherently nonlinear responses in the relationship by allowing for separate regimes.

5. Conclusions

The extensive literature on the Federal Reserve System's use of the discount rate as a policy instrument illustrates that the interaction of this policy tool with other instruments and indicators of policy is a complex phenomenon. The constraints placed by Fed policymakers on their own use of the instrument – effectively ruling out frequent or contradictory motion of the discount rate – give rise to an interesting empirical framework in which the constrained nature of the instrument must be explicitly taken into account to capture the dynamic relationship between the discount and funds rates. Examination of this relationship over the October 1979 – 1995 period of a reserve target operating procedure lends considerable support for its representation as a threshold cointegrating system.

We see opportunities for improving this research by refining both the analytical framework and the empirical methodology. Analytically, it is preferable to model Fed policymakers' behavior by explicitly accounting for self-imposed constraints such as infrequent discount rate changes. Empirically, our testing strategies assume the equilibrium error process follows a linear stationary autoregressive process. Ideally these tests should reflect both the globally cointegrated nature of the relationship and the local behavior of threshold nonlinearity as an alternative hypothesis to the null of no cointegration and linearity. However, the class of possible threshold alternatives may be too large to make the formulation of such a test feasible. Furthermore, as discussed in Balke and Fomby (1994), the class of threshold models is not identified under the null, since the threshold parameters (θ) are nuisance parameters present only under the alternative hypothesis. Given these difficulties, the two-step procedure we use here breaks the analysis of the global cointegration relationship and its locally nonlinear behavior into two parts. In the first step, we test for linear cointegration, and, conditional on this, we try to identify the local behavior of the equilibrium error by testing for threshold nonlinearity. However, the identification of threshold values based on scatter plots can be criticised as an ad hoc, subjective process. Alternative testing procedures are under development by Andrews (1993), Bai and Perron (1994), and Hansen (1996) and may be extensible to the two-threshold framework. The theory underlying these tests is, however, embryonic, while Tsay's procedures provide a feasible and justifiable alternative today.

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Table 3
Stationarity Tests for Weekly Discount Rate and Federal Funds Rate

Test	Oct. 1979 – Jan. 1996	Oct. 1979 – Oct. 1982	Nov. 1982 – Jan. 1996
Discount Rate			
ADF(8)	-1.67	-2.07	-1.26
PP(8)	-1.22	-1.34	-1.88
ADF(8) w/trend	-2.14	-2.03	-0.99
PP(8) w/trend	-1.47	-1.36	-0.88
KPSS(8) level	7.59 §	0.24	5.45 §
KPSS(8) trend	0.79 §	0.24 §	0.39 §
Federal Funds Rate			
ADF(8)	-2.33	-2.63	-1.29
PP(8)	-3.01 §	-4.04 §	-3.03 §
ADF(8) w/trend	-3.78 §	-2.55	-1.48
PP(8) w/trend	-6.37 §	-4.01 §	-5.35 §
KPSS(8) level	6.37 §	0.22	4.57 §
KPSS(8) trend	0.45 §	0.21 §	0.41 §

Notes: § denotes significance at the 95 per cent level of confidence. ADF is the augmented Dickey-Fuller test with 8 lags, for which the 95 per cent critical value is -2.86 (-3.41 with trend). PP is the Phillips-Perron test with 8 lags, with the same critical values. KPSS is the Kwiatkowski et al. (1992) test, with a null hypothesis of stationarity; the 95 per cent critical values are 0.463 for levels and 0.146 with trend. The samples contain 842, 160, and 682 weekly values, respectively.

Table 4
Threshold Values of the Equilibrium Error, Oct. 1979 – Jan. 1996

	Lower Threshold Values					
	-0.2587	-0.4433	-0.4887	-0.5287	-0.5772	-0.6541
number of weeks	349	241	219	208	189	161
mean spread	18	5	3	0	-5	-14
median spread	26	21	21	20	14	8

	Upper Threshold Values			
	0.3435	0.4466	0.4682	0.7520
number of weeks	248	207	200	156
mean spread	244	266	270	300
median spread	208	228	233	260

Notes: these threshold values were obtained by analysis of recursive "t" ratio plots and series generated using the Tsay (1989) methodology. The number of weeks are those weeks when the equilibrium error fell below (above) the lower (upper) threshold value. The mean and median spreads are the difference between the Federal funds rate and the discount rate during those weeks, in basis points.

Table 5
Estimated Band-Threshold Autoregressive Model for the Equilibrium Error, Oct. 1979 – Jan. 1996

	Regime		
	Lower	Middle	Upper
Threshold Variable	$z_{t-7} < -0.2587$	$-0.2587 < z_{t-7} < 0.3435$	$z_{t-7} > 0.3435$
Number of weeks	347	241	246
Lags of dep.var.	3	2	2
R-squared	0.545	0.347	0.549
R-bar-squared	0.541	0.342	0.545
Std. Error of Est.	0.739	0.657	1.065
Q statistic	39.52	2.96	34.03
Tail probability	0.32	1.00	0.56
Akaike criterion	1757.79	955.62	1223.25

Table 6
Estimated Vector Error Correction Model for the FF / DR Relationship, Oct. 1979 – Jan. 1996

	Regime		
	Lower	Middle	Upper
Threshold Variable	$z_{t-7} < -0.2528$	$-0.2528 < z_{t-7} < 0.3435$	$z_{t-7} > 0.3435$
		Dependent Variable: ΔFF_t	
F-test for ΔFF_t	17.01 (0.00)	23.85 (0.00)	192.22 (0.00)
F-test for ΔDR_t	8.77 (0.00)	2.34 (0.10)	22.29 (0.00)
z_{t-1}	-0.161 (0.061)	-0.282 (0.068)	-0.077 (0.047)
$\overline{R}^2$	0.276	0.359	0.553
Std. Error of Est.	0.891	0.503	0.956
		Dependent Variable: ΔDR_t	
F-test for ΔFF_t	0.83 (0.48)	4.49 (0.01)	28.48 (0.00)
F-test for ΔDR_t	0.97 (0.41)	1.10 (0.33)	0.045 (0.83)
z_{t-1}	0.040 (0.01)	0.048 (0.019)	0.033 (0.006)
$\overline{R}^2$	0.096	0.084	0.121
Std. Error of Est.	0.147	0.141	0.127
Lag order	3	2	1
Number of weeks	347	242	246

Note: F-tests pertain to the hypothesis that all lags of the variable may be excluded from the equation. Tail probabilities are given in parentheses. Estimated standard errors for z_{t-1} coefficients are given in parentheses.