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James E. Anderson
Boston College,

Will Martin
World Bank,

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James E. Anderson — Boston College

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Abstract

A simple general equilibrium model of an economy with distortionary taxes and public goods is used to extend, unify and clean up the welfare analysis of changes in taxation, redistribution and the provision of public goods. We clarify the distinction between compensation and money metric measures of the welfare impact of fiscal changes and show that the equivalent variation measure dominates other measures. We provide an integrated approach to marginal tax and public good changes when public goods have real resource costs and must be financed by distortionary taxation using the concepts of the marginal cost of funds, the fiscal price of public goods and the virtual price of public goods. Here too, the compensation version of these concepts dominates the money metric version.

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Correspondence to: James E. Anderson, Dept. of Economics, Boston College, Chestnut Hill MA 02167

The Welfare Analysis of Fiscal Policy

The literature on the welfare impact of tax changes and public goods changes is a formidable thicket of different methods (primal vs. dual), results from special cases (initial undistorted equilibrium, lump sum distribution, additively separable public goods) and loose claims about approximation. There are distinct and consequential differences in the methods advocated in the literature, with no obvious reason to select one over the others. This paper sets out a model of welfare evaluation in a representative consumer economy with all the basic elements of fiscal policy active. By developing a simple dual framework, we are able to clear up a great deal of confusion and error in the literature, and provide firm recommendations for both marginal and discrete policy analysis. We show that one approach is dominant. Clean fiscal accounting should proceed with the equivalent variation method.

The first purpose of the paper is to provide a map between four distinct and well-known measures of the welfare effects of fiscal changes and use it to show that one measure is superior. The four fundamental measures are set out in a 2x2 table, classified according to the reference point (initial situation vs. new situation), and according to the type of measure: compensation vs. utility. *Compensation measures* calculate a sum which could be extracted following the switch in policy while still supporting the reference utility and satisfying all budget constraints. *Money metric of utility* measures convert the change in equilibrium utility to a change in expenditure using the representative agent's expenditure function, with the reference point being either old or new prices. The compensation and money metric measures are fundamentally distinct even for infinitesimal changes when the economy is initially distorted.

One measure is superior to the other three. The equivalent variation is a compensation measure which is also a utility index, or a *compensation metric of utility.* In contrast to the other compensation measure, the compensating variation, the equivalent variation can always transitively order a set of fiscal plans.[1] In contrast to the money metric measures, the equivalent variation for a distorted economy measures a sum which could be extracted following a switch in policies while maintaining general equilibrium, while the money metric measures generally do not equal either compensation measure. We argue that the compensation property of the welfare measure can be critical in international or inter-regional comparisons of the payoff to fiscal projects, which arises naturally in the allocation of funds from the World Bank to client countries or from federal central governments to subsidiary governments. (We recognize that the 'interpersonal' comparison of utility across nations is involved here, which may require differential weighting of national equivalent variations.)

Using the same structure we also show that two additional measures of welfare change advocated in the literature (e.g. by Mayshar (1990)) are equivalent to the money metric measures, and hence are dominated by the equivalent variation. The analysis is carried out in Section I.

The second purpose of the paper is to provide a general model of marginal tax change and of public good supply change or redistribution undertaken with the tax revenues. The model allows us to sort out both confusion and error in the literature in the use of key concepts such as the Marginal Cost of Funds (MCF) and the closely related Marginal Excess Burden (MEB). We show that the fundamental distinction is between compensation and money metric versions of the concepts, and mixing them up has led to occasional error. For the class of money metric measures, we identify a further important distinction between cases with exogenous distortionary taxes and cases with endogenous distortionary taxes. We show that for marginal changes in public goods, the key concepts are the fiscal price (the marginal fiscal cost) and virtual price (marginal willingness to pay) of public goods. The tool we use here is the dual method, extended to include the provision

[1]As is well-known, without restrictions on income effects, the compensating variation may be intransitive. See Chipman and Moore (1980).

of public goods[2] which must be financed by the government and generate benefits to consumers and/or producers. No special substitutability or separability restrictions for public goods are imposed.[3] A key advantage of our approach is an integrated evaluation of the cost of raising revenues and the benefit of spending those revenues. For both tax changes and public goods changes we show that compensation measures are superior because they are simpler and allow the analysis to be decomposed. This is in addition to the advantage of compensation measures for international or inter-regional comparisons. The marginal analysis is carried out in Section II.

Finally, we illustrate the potential empirical importance of the distinctions between the measures considered with some simple, simulations of stylized changes in taxes and in the provision of public goods supply changes. Our results also suggest the importance for empirical work of focusing on the tax revenue substitution effects of public goods supply, as these are typically twice the size of the tax revenue substitution effects of marginal taxation. This analysis is in Section III.

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[2]For the representative consumer economy we study, the usual distinction between public goods and public supply of private (rival and excludable) goods is empty, so we use the term public goods.

[3]Some of these points are made in Schöb (1994). However, he does not advocate compensation measures as we do, does not analyze the error arising from mixing the compensation and money metric measures, and in fact repeats the error himself. Moreover, he advocates a rather confusing measure of the ‘marginal benefit of public goods, equal to what in our terms is the ratio of the virtual price to the shadow price. We think this treatment is less clean than our decomposition, which permits the social cost of tax-financed public goods to be compared with the social benefit. See Section II.

I. Classifying and Ranking the Welfare Measures

We study tax and public goods (fiscal) policy in a closed static representative agent economy with a Ricardian technology . The simplifying assumption of fixed prices for all goods also arises in a small open economy, and where appropriate the trade model interpretation is applied.[4] Some commodity prices are distorted by taxation while others are assumed to be nontaxable. The numeraire is one of the nontaxable commodities. The social budget constraint (the sum of government and private budget constraints) is equivalent to the balance of trade constraint in a small open economy setting.

Compensation measures give the social budget surplus or net foreign exchange released by switching between the old and the new fiscal policy while maintaining the level of utility at either its old or new reference level, and still meeting the social budget or balance of trade constraint. *Money metric utility* measures are defined as the money required by the consumer to purchase the change in equilibrium utility due to switching from the old to the new policy (see Chipman and Moore (1980)). Money metric utility can be measured using the old or the new consumer prices as a reference.

The *equivalent variation* is the compensation measure based on using the new level of equilibrium utility as a reference. Because it uses the new level of utility as a reference, it is a metric of utility, like the money metric measures. However, unlike them, it is a compensation measure, whereas the money metric measures are not. The fiscal policy literature has ignored the fundamental distinction between the equivalent variation and the money metric measures, possibly because it has effect only in distorted economies, whereas the underlying consumer's surplus theory is typically developed with local comparative static analysis about initially undistorted equilibria.[5] For a distorted

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[4]Relaxing the fixed price assumption or including nontraded taxed goods would add nothing essential to the points we make while complicating the analysis in well-known ways.

[5]For local changes about an initial undistorted equilibrium, the money metric utility measure defined on the old price base is equal to the compensation measure defined at the new level of utility and the term *equivalent variation* has been applied more or less interchangeably to both measures.

representative agent economy, the money metric adds to the equivalent variation a tax revenue change due to the shift in equilibrium utility. For this reason, while the money metric is a valid utility indicator, it loses any interpretation as a compensation measure.

In this section for simplicity we assume away public goods --- all taxation is redistributive, with revenue from distortionary taxes returned in a lump sum to the representative agent. (All our results ranking and relating the welfare measures generalize to the inclusion of public goods, switches among distortionary taxes, etc.) More consequentially, we also assume a representative agent economy. The representative agent is a price-taking optimizer with no market distortions other than those due to the taxes or subsidies. We analyze an exogenous change in distortionary taxes resulting in an endogenous 'revenue neutral' change in the lump sum tax or subsidy to private agents. The distortion vector t forces a difference between the fixed Ricardian producer price vector p* and the distorted price vector p, where p = p* + t. Evidently, a value of p implies a value of t, given fixed p*. For simplicity, in what follows we treat p as the tax instrument. Since p* is fixed, it is suppressed as an argument of the analysis. All tax revenues are collected and returned to the representative agent in a lump sum.

Our method of relating the four measures uses what we call the *social budget balance function,* which for the open economy is equivalent to the balance of trade function (see for example Anderson and Neary (1992)). The social budget balance function B(p,u) gives the sum which is required from outside the economy to support utility level u when the distorted price vector facing the representative consumer is p and general equilibrium prevails. A non zero value of B is interpreted as a gift to or from outside the system, from a hypothetical donor. In the small open economy interpretation the social budget balance function is the balance of trade function B(p,u), which returns the net foreign exchange required in general equilibrium to support a fixed utility level, u, when the distorted price vector facing the representative agent is p, differing by the trade tax/subsidy vector from the external price vector p*. We review the properties of B(p,u) below when required.

A. Four Welfare Measures

The four distinct measures of welfare can now be laid out. The class of *compensation measures* evaluates the surplus foreign exchange which could be extracted from (or must be given to) the economy when the tax policy is switched from its initial level to its new level or back again while simultaneously maintaining the utility at the reference level. This class of measures compares an actual equilibrium with a hypothetical 'compensated' equilibrium. The Compensating Variation (CV) is based on the initial equilibrium utility. The Equivalent Variation (EV) is based on the new equilibrium utility. The class of *money metric utility measures* compares the two actual equilibrium utilities (associated with the old and new tax policies) in terms of a money metric. Money metric Old (MO) is evaluated based on the initial prices while Money metric New (MN) is based on the new prices. These measures are collected in Table 1 with the basis noted following the measure. The definitions of the measures are given below.

Table 1. Classification of welfare measures

	Old	**New**
Compensation	CV (u_0)	EV (u_1)
Money Metric Utility	MO (p_0)	MN (p_1)

Within the class of compensation measures the *compensating variation* is:

(1.1) $CV = B(p^0,u^0) - B(p^1,u^0) = - B(p^1,u^0).$

The *equivalent variation* is:

(1.2) $EV = B(p^0,u^1) - B(p^1,u^1) = B(p^0,u^1).$

Note that these measures compare an actual equilibrium budget, either $B(p^0,u^0)$ or $B(p^1,u^1)$, with a hypothetical compensated equilibrium budget, either $B(p^1,u^0)$ or $B(p^0,u^1)$. They measure the amount of compensation which could actually be extracted or must be added to maintain the representative agent as well off as in the benchmark equilibrium. The second equality follows from the static equilibrium condition B = 0.

The class of money metric measures is based on the net (or for an open economy, the trade) expenditure function. The net expenditure function is defined as the difference between consumer expenditure and 'gross domestic product[6] at each price vector p and utility u. For a closed Ricardian economy the net expenditure function E(p,u) is equal to e(p,u) - g, where g is the supply of labor, the numeraire. For a small open economy, the trade expenditure function E(p,u) is equal to e(p, u) - g(p) where e(p, u) is the consumer's expenditure function for given domestic prices (p) and utility level (u) and the gross domestic product function g(p) is the maximum attainable production income given producer price vector p. In the trade model interpretation, g(p) need not reflect Ricardian technology since fixed prices p* are external, and the domestic price vector p includes the prices of traded commodities consumed by the household and supplied by the production sector.

The money metric utility measures use the transformation from utils to expenditure units which is embedded in the net expenditure function. Conditional on the cardinal representation of utility and the implied functional form of E, changes in E induced by changes in u trace out a money metric of utility. The money metric based on initial (Old) prices is equal to:

(1.4) $MO = E(p^0,u^1) - E(p^0,u^0).$

The money metric based on the New prices is equal to:

(1.5) $MN = E(p^1,u^1) - E(p^1,u^0).$

For both MO and MN, the change in utility is given a money value or metric via the expenditure function. Its interpretation is simple: MO (MN) gives the money required by

[6]The usage 'gross domestic product' function in the theoretical literature refers to the maximum value function which solves the optimal production program. It differs from the usage in the empirical literature, where the term refers to the value of product net of indirect taxes (gdp at factor cost) or production revenue plus tariff revenue (gdp at market prices).

the consumer to purchase the equilibrium rise in utility at the old (new) prices. Since the expenditure function is homogeneous of degree one in prices, it is also appropriate to evaluate the ‘money’ metric measures in terms of a numeraire.

The relation between all four measures is straightforwardly given by developing the properties of the social budget balance function. The social budget balance is equal to the sum of the net private expenditure and the net public expenditure. Net public expenditure is assumed to result in a lump sum transfer to the representative agent, so that the government budget deficit net of the transfer is equal to zero. The net public expenditure in the simple case of no government expenditures is negative, implying a lump sum subsidy to private agents. Thus,

(1.6) $$B(p,u) = E(p,u) - [p - p^*]'E_p(p,u),$$

where the term $[p - p^*]'E_p$ is net government revenue (using Shephard’s Lemma).

The CV measure is rewritten using (1.6) and $B(p^1,u^1) = 0$ as:

(1.7) $$\begin{aligned} CV &= E(p^1,u^1) - E(p^1,u^0) + [p^1 - p^*]' (E_p(p^1,u^0) - E_p(p^1,u^1)) \\ &= MN + [p^1 - p^*]' (E_p(p^1,u^0) - E_p(p^1,u^1)) \ . \end{aligned}$$

Similarly, we may derive

(1.8) $$EV = MO + [p^0 - p^*]' (E_p(p^0,u^0) - E_p(p^0,u^1)) \ .$$

For the small open economy case, the only difference is that p is the domestic price vector rather than the consumer price vector and E_p is the excess demand vector rather than the demand vector.

From (1.7) and (1.8) the four measures are distinct in general, but some equivalencies hold in special cases. For small changes in the taxes, MN = MO and CV = EV. For non zero taxes, it will not generally be true that MN or MO is equal to either CV or EV. These equalities obtain only if p^1 or p^0 is equal to p^* --- an undistorted solution. A focus on undistorted positions helps explain the tendency in the public finance literature to ignore the difference between compensation and money metric measures. For small changes, by (1.7) and (1.8) the four concepts are related by:

(1.9) $EV = MO - [p - p^*]'E_{pu}du = MO - t'e_{pu}du$

(1.10) $CV = MN - [p - p^*]'E_{pu}du. = MN - t'e_{pu}du.$

The second terms on the right hand side are income effects which induce changes in tax revenues. Unless t is equal to zero, where we deal with small changes about an initial undistorted equilibrium, the income effect vanishes only at du=0. In other words, it is a first order term which does not wash out in approximations.

More structure which links the compensation and money metric concepts is revealed from a welfare accounting for an infinitesimally small change in the tax vector, dp equal to dt. Totally differentiate the social budget balance function (1.6) and impose the equilibrium condition dB=0 to obtain:

(1.11) $(1 - [p - p^*]'E_{pu}/E_u)E_u du = - [p - p^*]'E_{pp}dp.$

The left hand side term $E_u du$ or, equivalently $e_u du$, represents an infinitesimal change in either money metric measure and the right hand side represents an infinitesimal change in either of the compensation measures. The change in money metric utility is obtained by dividing the compensation measure ($-B_p dp$) through by the term $(1- [p - p^*]'E_{pu}/E_u)$. The resulting term $1/((1 - [p - p^*]'E_{pu}/E_u)$ we call the *fiscal multiplier*, while in the open economy case it is called (see for example Anderson and Neary (1992)) the *shadow price of foreign exchange*. The fiscal multiplier gives the price of an additional unit of money metric utility in terms of (by means of) external compensation.

With positive taxes on normal goods, the fiscal multiplier will generally be positive and greater than one under well known conditions (Hatta, 1977). This also makes the money metric measures larger in absolute value than the compensation measures, by (1.9) and (1.10). If the income effects on relatively heavily taxed goods are negative (as is the case with endogenous labor supply, where leisure is in effect a subsidized normal good), then the fiscal multiplier may be smaller than unity, causing the money metric measures to be smaller than the compensation measures, a pattern which is evident in Fullerton (1991) and in Ballard and Fullerton (1992). Negative values of the fiscal multiplier are a curiosum, though possible.

B. Mayshar's Measures

Mayshar (1990) recently proposed two apparently *new* measures of welfare change designed to capture the excess burden of taxation. These measures combine partial equilibrium[7] CV and EV measures of the effects of taxation on consumers (and producers in the open economy case) with changes in *actual* taxation revenues. The primary justification offered for the use of these new measures is that the actual revenue raised by the taxation system, rather than a hypothetical measure of compensated revenue, should be used in evaluating the welfare costs of taxation.

The partial equilibrium version of CV, PCV, is equal to $E(p^0, u^0) - E(p^1, u^0)$. Mayshar's preferred measure (1990, p267) is the sum of PCV and the actual change in tax revenue. This is:

$$(1.12) \qquad E(p^0, u^0) - E(p^1, u^0) \quad + \quad t^1 E_p(p^1, u^1) - t^0 E_p(p^0, u^0)$$

Taking advantage of the fact that $E(p^i, u^i) - t^i E_p(p^i, u^i) = 0$ for $i = 0,1$; the Mayshar measure (1.12) may be rewritten as:

$$(1.13) \qquad E(p^1, u^1) - E(p^1, u^0) \quad = \quad MN,$$

the money metric measure based on new prices defined in equation (1.5). Mayshar's alternative measure using the partial equilibrium equivalent variation PEV to measure the effect of taxation on consumers can be transformed in exactly the same way to yield a money metric measure of utility change based on old prices, MO.

Thus the measures of the welfare burden of taxation proposed by Mayshar are simply reformulations of the well known money-metric measures of utility change. The Mayshar formulation (1.12) provides some additional insight into the interpretation of the money metric measures with its decomposition into effects on net expenditure changes and

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[7]Partial equilibrium in the sense that they do not account for the loss in government revenue transferred back to the representative agent.

tax revenue changes. However, it need not contribute to the counter-productive proliferation of distinct measures in this field.

C. The Choice of Measures

Which measure should be used? We argue here for the dominance of the equivalent variation, in contrast to some of the recent literature.

Ballard and Fullerton (1992, and in their earlier work) argue for money metric measures. Mayshar argues for what turn out to be money metric measures on the grounds that they reflect the actual change in tax revenue. Kay (1980) argues for the equivalent variation on the grounds that it reflects changes in actual tax revenue, but this only holds if the initial situation is undistorted. McKenzie (1983, p37) argues for MO on the grounds that it allows comparison of a range of options (alternative p^1 vectors), and ensures an equal ranking of all outcomes for which a representative consumer would be indifferent. Auerbach (1985, p93) is agnostic but argues for the use of the equivalent variation measure if the objective is to minimize excess burden through optimal taxation, on the grounds that at the optimal tax point, this function is evaluated at the actual tax point, rather than a hypothetical tax point. This advantage would carry over to the money metric at new prices (MN), but not to the MO measure advocated by McKenzie.

We argue that the EV is superior because it is both a compensation measure and a compensation metric of utility. The economic distinction between CV, EV and the money metric MN, MO measures lies in the fact that EV (CV) is a surplus which could *actually be extracted* with p^0 (p^1) and leave the consumer supported at u^1 (u^0). The money metric MN (MO) has no such interpretation, since it adds to that surplus a difference in revenues which arises due to the shift in equilibria. To see this we proceed in two steps. The equilibrium level of utility u^1 is solved from the social budget balance requirement $B(p^1,u^1)=0$. This requirement defines the *fiscal utility function:*

$$v(p) \quad = \quad \{u \mid B(p,u) = 0\}.$$

Now replace u^1 with the fiscal utility function in the definition of EV in (1.2):

(1.14) $EV = B(p^0,v(p^1))$.

For any p^1 such that the shadow price of foreign exchange remains positive, the equivalent variation function $B(p^0,v(p^1))$ is a metric of utility as well as a compensation measure.

The equivalent variation dominates money metric measures whenever its compensation property is likely to be important. This prominently includes international or interregional comparisons of changes in fiscal regimes. This arises naturally in the context of international transfers by the World Bank and the IMF, or of transfers to subsidiary jurisdictions by national central governments, both accompanying a package of fiscal changes. In analyzing the benefit of a set of such transfers combined with fiscal changes, a natural tool is the Kaldor-Hicks compensation principle. To apply it, the donor must start with consistently calculated compensation metrics for each nation or region. Formally, let the international or central government inward transfer β be defined for region k by $\beta^k = B^k(p^{k0},u^{k1})$. β^k is the amount of the transfer needed at unchanged fiscal policy to obtain as much utility in region k as the new fiscal policy obtains. Then $\Sigma\beta^k$ measures whether the set of policies $\{p^{1k}\}$ improves aggregate efficiency, potential gainers in the new position being able to compensate potential losers. The equivalent variation is also the natural building block for a social welfare function approach because it simplifies application of the international or interregional budget constraint. Let social welfare W be defined over the set of equivalent variation functions by:

$$W = F(\{B^k[p^{k0},v(p^{k1})]\}) = F(\{\beta^k\}).$$

Reallocation by the central fiscal authority or international agency is equivalent to reallocating $\{\beta^k\}$ subject to the budget constraint that $\Sigma\beta^k$ remain constant. Welfare rises for $\{d\beta^k\}$ such that $\Sigma F_k d\beta^k > 0$, $\Sigma d\beta^k = 0$; where F_k is the social marginal utility of a dollar transferred to k. Without denying the essential difficulty of the interpersonal comparisons of utility (swept under the rug by the representative agent model within fiscal jurisdictions) embedded in F or $\{F_k\}$, it is clear that such comparisons must regularly be made by international donors or lenders at subsidized rates, and within nations by central governments transferring among local jurisdictions.

The advantage of being a compensation metric of utility does not carry over to the compensating variation measure. (Homotheticity of preferences is sufficient for a transitive ranking of preferences is to be preserved by the CV). However, as is well-known, CV and EV are identical for small changes and for most practical purposes CV is often easier to calculate and is usually a close approximation to EV.

II. Marginal Fiscal Accounting with Public Goods and Redistribution

Now we consider a system of fiscal accounting for marginal changes in taxation, redistribution and public goods without any special assumptions about the initial situation or about the substitutability between the public and private goods. We generalize and extend the public finance literature (e.g., Auerbach 1985) and the cost-benefit literature (e.g. Fane 1991) to allow public goods to cost something to produce, and to interact with various fiscal distortions present in the economy. This general system allows us to clarify the concepts of the Marginal Cost of Funds and what we call the fiscal price of public goods and establish useful relations between them. We draw a distinction between compensation and money metric versions of these concepts, while for small changes, there is no difference between EV and CV, or between MO and MN. We argue for the superiority of the compensation version due to its simplicity and usefulness in the decomposition of the analysis. We also straighten out a confusing analysis of the same issue by Fullerton (1991) and Ballard and Fullerton (1992). Because our purpose is in part pedagogic, we present an analysis of six fiscal changes which are considered in the literature.

Public goods are denoted by the vector G. The elements of G which are positive must be goods for which the government is the only source of supply, or for which the government can determine the available supply[8]. Thus the representative agent's virtual price or marginal willingness to pay for a unit of G is determined by the supply of G. The constrained net expenditure function E(p,G,u) gives the net expenditure on the privately provided goods which are not subject to the quantity constraints imposed on provision of G. As previously, E is the difference between the representative consumer's constrained expenditure e(p,G,u) and gross domestic product, which includes the value of production

[8] This situation differs from the conventional case considered in most treatments of cost-benefit analysis, where government provided goods are also available at a price determined by world market prices and any trade distortions.

of public goods if any. For a Ricardian economy, labor is the natural numeraire and g is equal to the supply of labor.[9] Increases in elements of G which are valued by consumers will lower the cost of achieving any given level of utility. Thus, $E_G = e_G$ is negative and equal to minus the virtual price, or marginal willingness to pay vector of the representative agent. The vector G was understood to be inactive in the previous section and hence suppressed in the expenditure function and related derivative functions. See Anderson and Neary (1992) and Neary and Roberts (1980) for the properties of the constrained net expenditure function. The marginal cost vector for the public goods is equal to π^*, reflecting either Ricardian production or external supply at fixed price. Revenue is raised by consumption taxation (or trade taxation) at specific rate t, equal to p - p*. A fiscal change is a set of changes in public goods dG, financed by a set of tax changes dt (equal to dp) or by a change in the amount of lump sum revenue raised from or returned to the representative consumer, or by an external transfer $d\beta$ (the unbalanced budget case).

To incorporate these additional features, detailing the government budget, we need to extend the basic definition of the social budget balance function given in (1.6) and decompose it into government and private budgets. Let R denote the net government transfer to the private sector. The resulting setup for balanced budgets is:

(2.1) $\pi^{*\prime}G + R - [p - p^*]'E_p(p,G,u) = 0$ *government budget constraint*

(2.2) $E(p, G,u) - R = 0$ *private budget constraint.*

The redistribution R is the amount of revenue collected and returned as a lump sum to the consumer. (R can of course be negative, implying lump sum taxation.) The social budget balance function B(p,G,u) is defined by solving for R and substituting (2.1) into (2.2).

(2.3) $B(p,G,u) = E(p,G,u) + \pi^{*\prime}G - [p - p^*]'E_p(p,G,u).$

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[9]For the small open economy interpretation it is convenient to assume that public goods may also be externally purchased at fixed price vector π^*. For the small open economy case with trade taxes, production need not be Ricardian, and gross domestic product is equal to $g(p,\pi^*)$, which is invariant to G. The invariance of g to G arises because marginal supply is from the world market.

Equilibrium utility is determined by setting B equal to zero. Fiscal policies change p, G and R subject to the government budget constraint (2.1) (implying a dependency among changes in the three categories of fiscal policy), with the social budget constraint yielding the change in u.

While the consolidated social budget constraint (2.3) is a convenient summary, it masks the interdependence of fiscal policy which is required by the government budget constraint (2.1) separately from the private budget constraint (2.2). Thus our accounting system is based on (2.1)-(2.2). Totally differentiating (2.1)-(2.2) with respect to G,p,R and u we obtain:

(2.4) $[\pi^{*\prime} - (p - p^*)'E_{pG}]dG + dR - [E_p' + (p - p^*)'E_{pp}]dp - (p - p^*)'E_{pu}du = d\beta$

(2.5) $E_G'dG - dR + E_p'dp + E_u du = 0.$

Here, $d\beta$ is an external transfer to the government. The accounting system implied by (2.4)-(2.5) is clarified by reducing the system through imposing special cases and through collecting and labeling various terms. We consider six cases emphasized in the literature. For two unbalanced budget cases we allow the right hand side of (2.4) to equal $d\beta$ instead of zero.

For the <u>tax reform case</u> (1) with dp exogenous and dR endogenous while dG = 0 and $d\beta = 0$, equation (1.11) is obtained by substituting the solution for dR from (2.4), the differential of the government budget constraint, into (2.5), the differential of the private budget constraint.

For <u>transfer-financed tax reform</u> (2) we derive the important concept of the Marginal Cost of Funds (MCF) in its compensation version. For this case the differential of the government budget is equal to the external transfer $d\beta$. The transfer is used to finance an endogenous tax reduction dp, while dR = 0 and dG = 0. The change in the government budget deficit (2.4) is:

$$d\beta = - [E_p' + (p - p^*)'E_{pp}]dp - (p - p^*)'E_{pu}du.$$

There are many tax changes which $d\beta$ can finance, so to restrict them for simplicity we impose the uniform surcharge assumption $dp = pd\tau$, where $d\tau$ is a scalar[10]. Then the endogenous change in the surcharge is

$$d\tau \quad = \quad \frac{1}{E_p'p + (p - p^*)'E_{pp}p}\, d\beta + \frac{1}{E_p'p + (p - p^*)'E_{pp}p}(p - p^*)'E_{pu}du.$$

Substituting for $dp = pd\tau$ into the private budget constraint (2.5) and isolating terms in du we have:

(2.6) $\left(1 - MCF(p - p^*)'X_I\right) E_u du/d\beta = MCF,$ where

$$MCF \equiv \frac{E_p'p}{E_p'p + (p - p^*)'E_{pp}p}.$$

MCF, the Marginal Cost of Funds is by (2.6) the compensated (utility constant) marginal cost of raising another dollar for external transfer (or to be thrown away or absorbed in graft) by means of commodity taxation. MCF is useful because it appears in many other more realistic fiscal experiments. MCF is ordinarily greater than one.

For the redistribution case (3) the government raises one dollar of lump sum taxation and uses it to replace one dollar's worth of revenue from distortionary taxation. Here, $dG= 0$, $d\beta = 0$, dR is exogenously set at 1 and dp is endogenously determined with $dp = pd\tau$. This fiscal experiment leads to the concept of Marginal Excess Burden (MEB). Following the same steps leading to (2.6), we obtain:

(2.6') $\left(1 - MCF(p - p^*)'X_I\right) E_u du/dR = MCF - 1 \equiv MEB.$

For the tax financed public goods project case (4) of $dG>0$ financed by and endogenous $dp>0$ while $d\beta = 0$ and $dR = 0$, the analysis identifies a key concept we call the *fiscal price of public goods.* The first term multiplying dG in the government budget constraint (2.4) is equal to the row vector of fiscal prices of public goods, γ' --- the sum of the marginal production cost vector $\pi^{*\prime}$ and minus the marginal tax revenue change due to the expansion of public goods supply, $-(p - p^*)'E_{pG}$. E_{pG} is negative if public and private

[10] A more realistic package of tax changes can be treated with a simple generalization. Let W denote a diagonal matrix of (positive or negative) weights and let $dp = Wpd\tau$. To generalize the expressions which follow to include this case, simply replace the post-multiplying vector p with the post-multiplying vector expression Wp.

goods are substitutes in a natural sense, hence the fiscal price will exceed the production cost. If public and private goods are not all substitutes, it is possible that the marginal tax revenue change could be positive, and the fiscal price could be lower than the production cost. See Anderson and Neary (1992) for a discussion of properties of E_{pG}. Notice that no separability assumptions about G are being utilized here, unlike much of the public finance literature. Our term fiscal price appears to be new though the formula is well known. Next, we consider the endogenous terms in dp. The need to finance the project dG drives the terms in dp for the balanced budget case. As in the redistribution case, there are many possible tax vectors which will do, so we reduce the dimensionality of the problem by restricting attention to equiproportional tax surcharge changes, $dp = pd\tau$. Solving for $d\tau$ in (2.4) with dR = 0,

$$d\tau = \frac{1}{E_p'p + (p - p^*)'E_{pp}p}\left(\gamma'dG - (p - p^*)'E_{pu}du\right) .$$

Substituting into (2.5) and rearranging we obtain:

$$(2.7) \quad \left(1 - MCF(p - p^*)'X_I\right) E_u du = \left(\pi - MCF\gamma\right)'dG.$$

Here, the term E_G in (2.5) is equal to minus the virtual price (marginal willingness to pay) vector π. Equation (2.7) is remarkably clean and simple in its intuition. Welfare is increasing in public goods expansion of good i if π_i, the private marginal willingness to pay for dG_i, exceeds the fiscal price of government expenditure γ_i (the marginal funds required for dG_i) times the marginal cost of funds. The term π - MCFγ may be called the virtual premium of public goods.

An alternative interpretation of (2.7) uses the concept of the *shadow price of public goods*, the vector of marginal net benefits of a gift of one unit of the public goods: $\sigma = [\pi + MCF(p - p^*)'E_{pG}]$.[11] The literature on project evaluation emphasizes such shadow prices. In σ, the virtual price benefit π is adjusted by the induced change in tax receipts

[11]The fiscal experiment yielding this shadow price involves $dp=pd\tau$, and no change in G apart from adding the gift of one unit of public good i, dg_i. Solve the government budget constraint for $d\tau/dg_i$ and substitute into the private budget constraint to obtain the expression in the text.

times the MCF for the distortionary taxation needed to recover the tax revenue. Our expression provides an operational way to implement Squire's (1989) insight that the compensated shadow price needs to be adjusted when taxes are distortionary. Substituting with σ, the right hand side of (2.7) is alternatively written as $(\sigma - MCF\pi^*)'dG$. The term $(\sigma - MCF\pi^*) = (\pi - MCF\gamma)$ may be called the shadow premium of public goods.

Tax-financed public goods also often include *user charges*, which are feasible lump sum taxes at the margin so long as the good is excludable and the charge is less than the virtual price. Suppose first that users are charged $(1-s)\pi^*$, $s \leq 1$. This means that the lump sum transfer in (2.4) is $dR = -(1-s)\pi^{*\prime}dG$, which yields

(2.7') $$\left(1 - MCF(p - p^*)'X_I\right) E_u du = \left(\pi - MCF\gamma\right)'dG + MEB(1-s)\pi^{*\prime}dG.$$

The second term reflects the marginal benefit of replacing distortionary taxation with $(1-s)\pi^{*\prime}dG$ worth of nondistortionary taxation. Alternatively, suppose that user charges offset *all* revenue changes in (2.4), permitting $dp = 0$. (This may not be feasible, depending on the value of π, the virtual price.) Solving (2.4)-(2.5) we obtain:

(2.7'') $$\left(1 - (p - p^*)'X_I\right) E_u du = \left(\pi - \gamma\right)'dG = \left(\tilde{\sigma} - \pi^*\right)'dG,$$

where $\tilde{\sigma}' = \pi' + (p-p^*)'E_{pG}$, the shadow price of public goods for the case where at the margin lump sum user charges are substituted for all otherwise necessary distortionary tax changes. For this special limiting case we obtain Warr’s (1977) result that projects financed with user charges are beneficial if profitable at shadow prices, without regard to the MCF.

From the transfer financed public goods project case (5)we derive the concept of Marginal Benefit of Funds (MBF), parallel to MCF. Here, an external transfer $d\beta$ is used to pay for a vector of public goods dG while $dp = 0$ and $dR = 0$. To restrict the vector of public goods changes we impose proportionality: $dG = Gd\beta$.[12] Collecting terms in du on the left hand side as previously we obtain

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[12]As in the tax vector change case, this specification can be generalized to allow for general public goods vector changes using a weighting matrix., including allowance for public projects with both output and input changes.

(2.8) $\left(1 - MBF(p - p^*)'X_I\right) E_u du/d\beta = MBF$, where

$$MBF \equiv \frac{\pi'G}{\gamma'G}.$$

We may relate (2.8) to (2.7) by noting that at constant u, the total revenue required for dG is $d\alpha = \gamma'Gd\beta$. If provided from external sources $d\alpha$ produces benefit equal to MBF. The right hand side of (2.7) is thus (MBF - MCF)$d\alpha$, the revenue requirement times the gap between the marginal benefit of external funds spent on public goods and the marginal benefit of external funds spent on a dollar of distortionary tax reduction.

Finally, for the <u>differential tax case</u> (6) the fiscal experiment is to alter one group of distortionary taxes exogenously and endogenously alter another group of distortionary taxes so as to maintain a constant amount of tax revenue. This analysis yields the concept of MCF_i for a group of goods i. Let the vector p be partitioned into the vectors p_1 and p_2. The second set of goods has an exogenous change in taxes $dp_2 = p_2 d\tau_2$. We solve the differential of the government budget constraint (2.4) for $dp_1 = p_1 d\tau_1$ with dG equal to zero and dR equal to zero. The solution for $d\tau_1/d\tau_2$ is

$$\frac{d\tau_1}{d\tau_2} = \frac{-1}{E_1'p_1 + (p - p^*)'E_{\bullet 1}p_1} + \frac{-1}{E_1'p_1 + (p - p^*)'E_{\bullet 1}p_1}(p - p^*)'E_{pu}\frac{du}{d\tau_2}.$$

Here, we denote E_i as the derivative of E with respect to p_i, E_{ij} as the substitution effect sub matrix of the vector of net demands E_i with respect to the vector of prices j and $E_{\bullet i}$ as the partitioned matrix $\begin{pmatrix} E_{1i} \\ E_{2i} \end{pmatrix}$. Substituting the solution for $d\tau_1/d\tau_2$ into the differential of the private budget constraint (2.5) with dG = 0 and dR = 0:

(2.9) $\left(1 - MCF_1(p - p^*)'X_I\right) E_u \frac{du}{d\tau_2} = \left(\frac{MCF_1}{MCF_2} - 1\right) E_2'p_2.$

In equation (2.9), MCF_i is naturally defined as

$$MCF_i \equiv \frac{E_i'p_i}{E_i'p_i + (p - p^*)'E_{\bullet i}p_i} \quad i = 1,2.$$

Equation (2.9) is intuitive: if the group 1 taxes are more costly, an increase in group 2 taxes coupled with a revenue neutral cut in group 1 taxes will be welfare improving.

Each of the cases above (tax reform, redistribution, unbalanced budget, public goods project and differential taxes) results in a *compensation measure* on the right hand side of (1.11), (2.6), (2.6'), (2.7) and its variants, (2.8) or (2.9). We can alternatively derive a *money metric measure* for each case, by multiplying through by the relevant fiscal multiplier to isolate $E_u du$. These multipliers differ significantly depending upon the fiscal experiment. The multiplier term in (2.6)-(2.7) differs from the multiplier term in the parametric tax change case (1.11) because with endogenous distortionary taxes, the tax revenue change due to the endogenous change in utility, $(p - p^*)'E_{pu}$, must be made up from tax changes which result in a cost to the social budget at the rate of the MCF. In (2.8), MBF pre multiplies the income effect term because in the experiment analyzed, distortionary tax changes are equal to zero and the endogenous revenue effect must be offset by endogenous public goods supply changes, which cost the budget at rate MBF.

Two prominent money metric measures are those implied by the transfer financed tax reform and redistribution cases. Multiplying (2.6) through by the fiscal multiplier, the Money Metric version of the Marginal Cost of Funds (MMCF) is defined as

$$(2.10) \quad MMCF = \frac{MCF}{1 - MCF(p - p^*)'X_I}.$$

Ballard, Fullerton and Ballard and Fullerton advocate MMCF as a measure and emphasize that even in the case of taxes on perfectly inelastic goods (where the term $(p - p^*)E_{pp}$ is equal to zero) , MMCF will generally not be equal to one although MCF is equal to 1. Finally, multiplying in (2.6') through by the fiscal multiplier, the Money metric version of MEB is defined as:

$$(2.11) \quad MMEB = \frac{MEB}{1 - MCF(p - p^*)'X_I}.$$

The development of MMEB in (2.11) clears up a confusion in the literature. Note that both MEB and MMEB are equal to zero in the case where taxation falls on goods with perfectly inelastic compensated demand. Ballard and Fullerton confuse this issue by calculating a version of money metric MEB as MMCF - 1, which may be less than zero. The procedure leads to the false conclusion that the money metric MEB can be positive when taxes are on inelastic goods. MCF arises in the fiscal experiment where an external dollar is donated to the government, permitting a one dollar reduction in distortionary tax revenue (the transfer financed tax reform case)[13], resulting in a compensated benefit of MCF. MMCF is obtained by multiplying by the fiscal multiplier. If, instead, the reduction of distortionary tax revenue must be raised from lump sum taxation (switching to the *redistribution case)* the compensated cost of the lump sum tax dollar is 1, and its money metric value is 1 times the fiscal multiplier, yielding the MMEB formula given in (2.11). Extending this point to other cases, it is critical to note from equation (2.7) that the fiscal multiplier applies to both the benefits and the costs of a particular government project. Specifically, the Money Metric version of the Virtual Price (MVP) of public goods is defined as

$$MVP = \frac{\pi}{1 - MCF(p - p^*)'X_I}$$

to be compared to the fiscal price γ times MMCF in the money metric version of (2.7). Thus, the substantial differences between the marginal excess burdens on a compensated and money metric (our terms) basis estimated by Fullerton (1991) have no policy implications, since in a proper analysis the benefit terms must also be consistently scaled either with the fiscal multiplier for money metric analysis or without it for compensation analysis. Ballard and Fullerton commit precisely this error in calculating MMEB as equal to MMCF - 1. Schöb (1994) repeats the error in defining the optimal (single) public good by equating the money metric MMCF to the ratio of the virtual price to the fiscal price, MBF for the single good. (The correct version of Schöb’s optimal condition is, by rearranging the right hand side of (2.7), MCF equal to π/γ.)

Based on our analysis of the six cases of fiscal experiments, it is important to distinguish between which fiscal instruments are endogenous and which are exogenous.

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[13]Something like this case may be associated with the World Bank Structural Adjustment Loan program in which loans are made to compensate for various adjustment costs associated with trade liberalization, including loss of government revenue.

Both the compensation measures and the fiscal multipliers change depending on which distortionary instruments are endogenous. The literature has distinguished between differential tax analysis (in which one tax is replaced by another of equal revenue yield) and balanced budget analysis (in which tax changes pay for public goods changes), a distinction which seems less useful. Confusingly, Ballard and Fullerton (1992) associate this difference with the quite separate distinction between compensation and money metric measures.[14]

A. Compensation vs. Money Metric Measures Again

We argue that the compensation version of all these marginal fiscal concepts is superior for three reasons. First, the compensation version is simpler than the money metric version. Second, the use of the compensation version of MCF aids greatly in a decomposition of the analysis; one which can save an analyst from errors like those of Ballard and Fullerton, and Schöb. Operationally a related advantage appears in applied project evaluation work, where estimates of MMCFγ are often taken from one study and compared with estimates of MVP taken from another study. The analysis above emphasizes that such money metric measures cannot be compared because they are likely to be based on different fiscal multipliers. The correction procedure involves first dividing both money metric measures through by the fiscal multipliers from each study (assuming these are reported) to obtain compensated measures, then possibly multiplying by a common fiscal multiplier if money metric measures are desired.

The most significant reason that compensation measures dominate money metric measures, as with the tax change case considered in Section I, is that they permit the

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[14]Ballard and Fullerton (p. 125) note that the balanced budget analysis (or Stiglitz-Dasgupta-Atkinson-Stern analysis) includes income effects, so that a lump sum tax in the presence of wage taxes, by increasing work effort, actually raises wage taxation, so that the marginal cost of funds is less than one. In contrast, the differential analysis (or Pigou-Harberger-Browning analysis) finds a marginal cost of funds equal to one.

There is a confusion sown in Fullerton (1991) and related papers by the practice of using differentials in revenue in the formulae to define versions of Marginal Excess Burden, as opposed to breaking changes in revenue into the components of change due to changes in u, p and G. (See Fullerton, p303.) Thus it is often difficult to make out exactly what Fullerton and others have in mind.

international or interregional comparability of fiscal projects which can be essential for the work of the hierarchic fiscal agencies such as the World Bank on a global level and central governments on a national level. MCF for a nation or region is its willingness to pay for a dollar of external transfer used to finance distortionary tax reductions. It is entirely sensible to rank potential clients on the basis of the size of MCF, the willingness to pay for a one dollar transfer. The money metric version of MCF does not have the willingness to pay interpretation, and thus fails to be internationally comparable. (The analyst must of course know the sign of the fiscal multiplier to apply the right sign to the compensation measure.)

Our analysis does *not* imply that fiscal multipliers are irrelevant, but only that they should be left out of marginal fiscal analysis. For discrete fiscal policy changes the fiscal multipliers are necessarily at work behind the scene in the calculation of the equivalent variation. EV in equation (1.14) requires as an input a calculation of the new level of utility based on the implicit fiscal utility function. Changes in utility due to fiscal changes necessarily involve the fiscal multiplier, as the analysis of this section shows.

III. Some Cobb-Douglas Simulations

This section will demonstrate that the difference between the money metric measures and the equivalent and compensating variations is numerically significant in some simple simulations of tax reform and of tax financed public goods supply. There is also some pedagogical value in presentation of the closed form solutions for various concepts which the Cobb-Douglas model permits

A. Tax Reform

For the tax reform case, the simplest model possible is chosen, in which a single good is subject to a tax. We shall consider two regimes: a low tax on a small expenditure share good and a high tax on a high expenditure share good. The first case is interpreted as a tariff for a small open economy specialized in production of its exports, which are untaxed. The second case is interpreted as a (market good) consumption tax in a closed Ricardian economy where the untaxed good is household production. Revenue is lump-sum redistributed to the consumer. Finally, the preferences of the representative consumer are Cobb-Douglas.

Let g be the gross domestic product, p* the fixed price (foreign supply price or Ricardian marginal cost in terms of labor) of the taxed good, and T be the tax factor, one plus the ad valorem tariff or consumption tax rate. g is a constant, as is a, the taxed good share parameter. The expenditure function of the representative consumer is

$$e = (p^*T)^a u,$$

and the tax revenue is equal to

$$p^*(T-1)a\frac{e}{p^*T} = (1-1/T)ae.$$

Then the social budget balance function is written as:

$$B(T,u) = (p^*T)^a u - g - (1-1/T)a(p^*T)^a u, \tag{2.1}$$

The fiscal utility function is solved from (2.1) as

$$v(T) = (Tp^*)^{-a}\frac{g}{1 - a + a/T} = u. \tag{2.2}$$

The equivalent variation function is formed by substituting $v(T^1)$ for u, and T^0 for T into equation (2.1):

$$(2.3)\quad EV(T^0;T^1) = \left(\frac{T^0}{T^1}\right)^a \left(\frac{1 - a + a/T^0}{1 - a + a/T^1}\right) g - g.$$

The compensating variation function is formed by substituting $v(T^0)$ for u and T^1 for T into minus equation (2.1):

$$(2.4)\quad CV(T^1;T^0) = -\left(\frac{T^1}{T^0}\right)^a \left(\frac{1 - a + a/T^1}{1 - a + a/T^0}\right) g + g.$$

The money metric in its new price basis form is formed by substitution of $v(T^1)$ for u and T^0 for T into the expenditure function $e = (p^*T)^a u$. This yields:

$$(2.5)\quad MN(T^0;T^1) = \left(\frac{T^0}{T^1}\right)^a \frac{g}{1 - a + a/T^1} - \frac{g}{1 - a + a/T^0}.$$

Finally, the money metric in its old price basis form is formed by substitution of $v(T^0)$ for u and T^1 for T into the expenditure function:

$$(2.6)\quad MO(T^1;T^0) = \frac{g}{1 - a + a/T^1} - \left(\frac{T^1}{T^0}\right)^a \frac{g}{1 - a + a/T^0}.$$

Table 2 below presents simulation of this simple model of welfare evaluation for a tariff regime case. Tariffs vary in a range of reasonable tariff values for a protectionist economy, from 10 per cent to 30 percent about an initial base of 20 per cent. The economy alternately has 10 per cent and 20 per cent of expenditure falling on imports: a reasonable range for a small open economy. The welfare measures are all scaled by GDP for comparability and expressed as percentages, hence all numbers in Table 2 are in per cents. The results clearly show the nonequivalence of the CV and EV vs. the expenditure function money metric measures. For normal goods, the money metric measures exceed the CV and EV in absolute value. However, the differences are small. Also, the results show the close approach of CV to EV and of MO to MN.

Table 2. Four Measures of Tariff Change (%)

	higher trade share (20%)		lower trade share (10%)	
tariff change	-10	10	10	-10
EV/GDP	0.1852	-0.265	-0.146	0.1027
CV/GDP	0.1848	-0.266	-0.147	0.1026
MN/GDP	0.1915	-0.275	-0.149	0.1044
MO/GDP	0.1882	-0.279	-0.150	0.1035

For larger tax burdens the difference between the concepts is not small when viewed as a percent of the EV measure. Consider a high consumption tax economy which taxes some forms of consumption, with the untaxed goods being interpreted as the consumption of household production. The base expenditure share subject to tax is assumed here to be equal to 50%. The base tax rate is alternately 40 and 60 per cent.

Table 3. Four Measures of Tax Change (%)

	Very high initial tax (60%)		High initial tax (40%)	
tax rate change	-50	50	30	-30
EV/GDP	2.657	3.914	-2.049	1.303
CV/GDP	2.589	-4.073	-2.092	1.287
MN/GDP	3.271	-4.817	-2.391	1.521
MO/GDP	2.712	-5.518	-2.634	1.348

As a percent of EV, the MN measure differs by about 25 per cent in the very high initial tax case and by about 20 per cent in the high initial tax case.

B. Changes in Public Goods Supply

We conclude with a simple simulation example of tax financed public goods supply, in which the difference between the two measures is again seen to be large, and in which the marginal concepts can be related. This example is also useful in showing how our methods might be applied in CGE models. The Cobb-Douglas example is extended to include a single public good G, assumed to be supplied at a price π^* equal to one. The constrained expenditure function for this case works out to be[15]

$$c(p,G,u) = (1-b)b^{b/(1-b)}\ p^{a/(1-b)}\ G^{-b/(1-b)}\ u^{1/(1-b)}.$$

Here, b is the expenditure share for the public good in the original unconstrained expenditure function $c(p,\pi,u)$, and a is the expenditure share for the taxed good. The consumer price of the taxed good, p, is equal to p*+t, with p* equal to unity. A third numeraire good is untaxed and left implicit, so a+b<1. The social budget balance function B(p,G,u) is built by substitution of (3.1) into (3.2) for the Cobb-Douglas case:

$$B(p,G,u) = c(p,G,u) - g + \{(-c_G)G - [p - p^*]c_p\}.$$

Based on this function and its derivative properties (c_p is equal to the demand for the taxed good and $-c_G$ is equal to π), we present fiscal experiments which reveal large differences between the compensation and money metric concepts. The money metric measures are based on natural extensions of MN and MO which use G^1 and G^0. The compensation measures are based on the social budget balance function, with

$$CV = -B(p^1,G^1,u^0)$$

$$EV = B(p^0,G^0,u^1).$$

The simulation is based the following parameters. The supply price of the consumption good is set at unity, the consumption good share a is equal to 0.5, the public good share b is equal to 0.2, the initial supply of the public good is equal to 10% of GDP, which for a balanced government budget requires a consumption ad valorem (equal to specific) tax rate equal to 19.048%. The first fiscal experiment is to raise the public good supply to 11% of GDP, covering the cost with a rise in the tax to 21.359%. The second

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[15] $c(p,G,u) = \max_h \{p^\alpha h^\beta u - hG\}$. The first order condition is solved for the virtual price h. Then the result is substituted into the maximand to obtain the text result.

experiment selects the optimal (subject to revenue raised by the consumption tax only) public goods supply. This requires an increase in public good supply to 17.857% of GDP, and an optimal consumption tax rate equal to 40%. (The first best optimal public good supply when lump sum taxation is available is equal to 20% of GDP.) All four measures in Table 4 show that the increase in public goods supply and taxes is beneficial. For all the cases, the money metric measures are larger than the compensation measures . For the smaller reform the differences amount to 11% to 12% of the compensation measure. For the optimal policy the differences amount to 11% to 21% of the compensation measure. Relative to GDP, the differences between money metric and compensation measures range from about a tenth of a per cent of GDP for the smaller reform to three-fourths of a per cent of GDP for the optimal policy. Thus the differences are substantial.

It is useful to relate the discrete results to the local analysis upon which our intuition is based. First, the money metric is locally equal to the corresponding compensation measure times the fiscal multiplier, by equation (2.7). Table 4 shows that for discrete changes this formula is inaccurate. For the 10% rise in public good supply case, EV/GDP (1.069%) times the fiscal multiplier (1.167) is equal to 1.234%, which is not very close to value shown for MO/GDP (1.188%). The local approximation does much worse for the larger move to the optimal policy.

Second, it is interesting to relate the results to the local analysis in equation (2.7) of the marginal cost of funds and the shadow premium of government supply. Evaluated at the new equilibrium, the last three lines of the table give the virtual price of the public good $\pi = -c_G$, its fiscal price $\gamma = \pi^* - tc_{pG}$, and the marginal cost of funds MCF = $c_p/(c_p+tc_{pp})$. At the new level of public good supply with a 10% rise in supply, the fiscal price is equal to 1.250 while the virtual price is equal to 2.273. The product of the marginal cost of funds (1.071) times the fiscal price is equal to 1.338, indicating that expansion in public goods supply is beneficial. At the optimal policy the shadow premium is equal to zero. The optimum is associated with a virtual price π equal to 1.400, a fiscal price of government supply γ equal to 1.250, and a marginal cost of funds MCF equal to 1.120.[16]

Finally, the results show that modeling the effect of public goods on tax revenue - tc_{pG} deserves at least as much attention from applied economists as modeling the MCF. At least for the plausible Cobb-Douglas example, $-tc_{pG}$ at 0.25, is more than twice as large as the marginal excess burden, MCF - 1, throughout the range of taxes shown. Without denying the difficulty of adequately treating c_{pG}, we believe public sector economists should try harder. A related point is made about quota treatment in Anderson and Neary (1992).

Table 4. Four Measures of Fiscal Change with Public Goods

	10% rise in G	optimal policy
tax	21.359%	40.000%
public good to GDP ratio	11%	17.857%
EV/GDP	1.069%	4.014%
CV/GDP	1.045%	3.507%
MO/GDP	1.188%	4.460%
MN/GDP	1.174%	4.269%
fiscal multiplier	1.167	1.389
virtual price of the public good	2.273	1.400
fiscal price of public good	1.250	1.250
marginal cost of funds	1.071	1.120

[16]The net marginal revenue requirement in the Cobb-Douglas case is constant. Using the Cobb-Douglas form:

$$c_p = \frac{a}{1-b} c/p \quad \text{and} \quad c_G = -\frac{b}{1-b} c/G.$$

Then $$c_{pG} = \frac{a}{1-b} c_G/p = c_p c_G/c = -\frac{b}{1-b} c_p/G.$$

Using the government budget constraint $tc_p = G$,

$$1 - tc_{pG} = 1 + b/(1-b) = 1/(1-b).$$

IV. Conclusions

Our study has identified four distinct types of money measures of welfare change for tax changes. These measures are differentiated by whether they are based on the compensation criterion, or on evaluation of a money metric for welfare change, and whether they are based on initial or final levels of utility (in the compensation case) or the price vector (in the money metric case). The widely accepted "hybrid" measures of welfare change based on hypothetical compensation measures and actual tax revenues were shown to be identical with the money metric measures, helping to reduce the confusing proliferation of distinct welfare measures in the field.

A fundamental choice is whether to focus on compensation measures or money metric measures. The choice cannot be fudged by appeal to absence of differences, as our numerical simulations show. The equivalent variation is both a compensation measure and a (compensation) metric of utility. Thus it is clearly superior for the broad class of examples in which the compensation property is likely to matter --- whenever there is need to compare results across economies. We recommend that in fiscal analysis based on Computable General Equilibrium models, the equivalent variation be used in place of either money metric. For partial equilibrium models, the compensating variation is frequently easier to calculate and often a close approximation to the equivalent variation, so it is a useful second choice.

The distinction between compensation and money metric types of measure also applies to the analysis of marginal changes in fiscal policy found in the literature. Here too we show that the compensation version of the marginal cost of funds and the virtual price of public goods is superior to the money metric version. We recommend that in applications, analysts always report the compensation versions of the MCF and the virtual price of public goods. Our analysis also allows us to sort out confusion and error in previous attempts to explain these concepts.

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