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A Theory of the Soft-budget Constraint

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Comments are welcome

Agency in Project Screening and Closing Decisions:  
A Theory of the Soft-budget Constraint

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Abstract

This paper studies the soft budget constraint problem in a principal-agent model. The agent screens projects and makes initial investment in the projects that have passed the screening. He then finds the types of the funded projects and decides to close some of the *ex post* inefficient ones among them. Closing projects sends an unfavorable signal about the agent's screening effort. Under the *ex ante* efficient contract, the agent has incentive to refinance some of the *ex post* inefficient projects.

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## 1. Introduction

Since Kornai's (1980) seminal work, it has been widely recognized that the problem of soft budget constraint is a very general and fundamental problem in socialist economies. Efforts have been made to explain the problem with varied emphases on political and economic institutions of socialism. Kornai himself attributes the problem to the "paternalistic" role of the socialist state. Bardhan (1993, p.151) noticed that "[w]henver the benefits of state policies of leniency...are concentrated and highly visible and their costs are diffuse, inevitably political pressure on the state, whether capitalist or socialist, builds to follow such policies." He suggested that the higher tolerance for mobility and unemployment explains harder budget constraint to business in capitalist than in socialist economies. Putterman (1993, p.157) emphasized "the non-decentralizability of ownership" in socialism that causes all monitoring and supervisory functions "to have to be filtered through the singular node of a people (or public)-state principal-agent relationship mediated by relations of political accountability, rather than by the exit mechanism of a competitive market." A common feature of these explanations is their emphasis on the political process for resource allocation in socialist economies, particularly the role of the state. The actual mechanism of how a political factor or state role leads to soft budget problem has been left not formally modeled.

Dewatripont and Maskin (1990) were the first to formally model the soft budget constraint phenomenon. Their model emphasizes the role of the financial institution in an economy and has the following key elements. The starting point of their theory is that lenders may have trouble at the outset in distinguishing between poor and good projects, poor projects being those that are *ex ante* unprofitable. However, even a poor project may be worth refinancing once major fixed costs have already been incurred. A centralized financial institution can make the *ax ante* threat that it will not refinance the poor project. However, the threat is not very credible because

refinancing is *ex post* efficient. When credit is sufficiently diffuse, refinancing has to come from a different lender. The prospect of sharing the benefit of the project with the second lender dampens the first lender's incentive to monitor the project, which may make the project not worthwhile at all. Decentralized credit is thus the way to create credibility in the threat that poor project will not be refinanced. The power of their theory extends to explaining differences in corporate finance found in different species of capitalist economies, such as the United States, Japan and Germany.

This paper is another attempt to understand the phenomenon of soft-budget constraint. The main question it tries to address is: Why in socialist economies are there firms not closed that seem to be in a long-term or even permanent money-losing situation?<sup>1</sup> A principal-agent model is built to study the informational effect of the agent's decision to close (not refinance) an on-going project and the principal's optimal incentive scheme for the agent that takes into account of the informational effect. There are two players in the model, the state as the principal and the planner as the agent. In the economy there are many projects, each requiring an initial and a subsequent investment. The projects have continuously distributed qualities, some of them *ex ante* (before the initial investment) profitable, others *ex ante* not but *ex post* (after the initial investment) profitable and still others unprofitable both *ex ante* and *ex post*.<sup>2</sup>

The planner's job is to screen out projects that are *ex ante* unprofitable and close (not refinance) projects that are *ex post* unprofitable. The planner can use his effort to obtain information about the qualities of the projects. The harder he works,

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<sup>1</sup>We take it as obvious that in socialist economies there are firms that are in a long-term or even permanent money losing position but still not closed and see it as a major part of the soft budget problem.

<sup>2</sup>Note that the projects that are *ex post* unprofitable are necessarily *ex ante* unprofitable also.

the more bad projects can be screened out. After the initial investment, the planner obtains perfect information about project qualities automatically (without effort). Suppose that the number of projects closed is public information. In making project closing decisions, the fundamental dilemma the planner faces is that every project closed is one he *ex ante* should have but failed to screen out.

The key question is then: Should the state's incentive scheme for the planner utilize the information of the number of projects closed? If yes, how? Conceivably, the state could write an incentive scheme that ties the planner's payoff to the observable final output (profit) only, ignoring completely the number of closed projects. This would induce the planner to close all projects that are *ex post* unprofitable. However, we show that, if the state's objective is to maximize the *ex ante* expected profit, any incentive scheme that completely ignores the number of projects closed is not *ex ante* optimal. The *ex ante* optimal incentive scheme requires instead that the information be used. Specifically, the way this information is used provides an incentive for the planner to close fewer projects than what is *ex post* efficient, creating the soft-budget problem observed in socialist economies. The rationale for this is found in the tradeoff between the cost of refinancing some of the *ex post* inefficient projects and the benefit of a higher *ex ante* effort by the agent. At the *ex post* efficient number of project closings the marginal cost of closing one fewer project is almost zero (second-order infinitesimal) but the marginal benefit is a nontrivial positive value (first-order infinitesimal) because closing one fewer project increases the agent's marginal return to effort by a nontrivial positive amount.

Our explanation of the soft-budget problem compliments those mentioned earlier, especially that of Dewatripont and Maskin, in fairly obvious ways. The model builds on Dewatripont and Maskin by starting with the *ex ante* information problem with project qualities and focusing on incentive problems in *ex post* refinancing

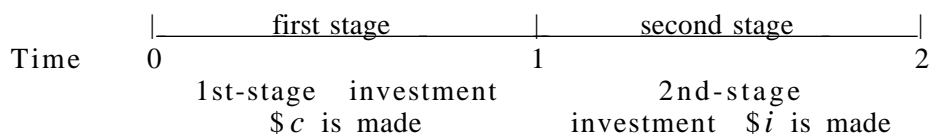
decisions. In Dewatripont and Maskin, the key institutional assumption is centralized credit in socialist economies. They showed that in socialist economies projects that are *ex ante* unprofitable but *ex post* profitable will be refinanced. Our model confirms this result, which is not surprising because, although implicit, our model assumes the same institution of centralized credit. However, our model moves beyond to show that even some *ex post* unprofitable projects will be refinanced. The new result is due to asymmetric information regarding the planner's effort besides the institution of centralized credit. The result provides an economic rationale for the observed paternalism or high tolerance of socialist state to long-term money-losing firms (or managers and workers in these firms) that has been explained for political and social reasons by other authors.

The plan for the paper is as follows. The model is introduced in Section 2. The main result of the paper is derived in Section 3. In Section 4, we discuss the renegotiation problem and two possible extensions to the model. Section 5 concludes the paper where we comment on the paper's contribution to the organization theory in general and the light it sheds on the debate of market socialism.

## 2. The Model

The principal in our model is the socialist state and the agent is the planner/bureaucracy. The state maximizes its net revenue, which is the gross revenue net of investment costs and the payment to the planner. The timing of the game is as follows.

The planner examines the projects      Types of projects revealed,      Returns realized,  
the projects      termination decision made      the planner rewarded



At time 0, the planner is given a continuum of projects and exerts an effort,  $a$ , to examine the projects. The disutility of his effort is  $d(a)$ .

The projects are indexed by  $\alpha \in [0,1]$  and offer an expected revenue of  $r(\alpha)$ , where  $r(\alpha)$  is a smooth increasing function. To realize the revenue, two stages of investments are needed. The investment is  $\$c$  in the first stage and  $\$i$  in the second stage. We call the difference between the gross revenue and the investment costs the value-added of a project.<sup>3</sup> A project is *ex ante* (at time 0) profitable if its expected value-added is non-negative, that is,  $r(\alpha) \geq c+i$ . Define the critical values  $\delta$  by  $r(\delta) = c+i$  and  $\xi$  by  $r(\xi) = i$  (see Figure 1). Projects in the range  $[\delta,1]$  are *ex ante* profitable; those in the range  $[\xi,\delta]$  are *ex ante* not but *ex post* (at time 1) profitable; and those in the range  $[0,\xi]$  unprofitable both *ex ante* and *ex post*.

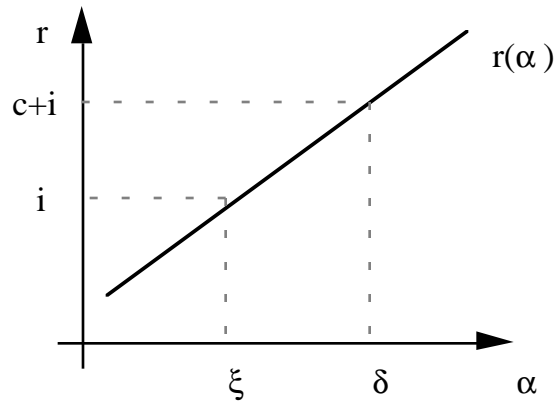


Figure 1

In the first stage, an investment  $\$c$  is made on projects that have passed the examination. Other projects are screened out and not funded.

We assume that all *ex ante* profitable projects and some *ex ante* unprofitable projects pass the examination and receive funding for the first-stage investment. The higher the effort, the fewer unprofitable projects pass. Specifically, We assume that projects in the interval  $[\alpha, \alpha + d\alpha]$  has probability  $p(a, \alpha)$  to be screened out, where  $p(a, \alpha) \in [0,1)$  for all  $a$  and  $\alpha$ ,  $p(a, \alpha) = 0$  if  $\alpha \geq \delta$ , and

<sup>3</sup>For the sake of simplicity, we assume that the interest rate is zero.

$$\frac{\partial p}{\partial a} > 0 \text{ for } \alpha < \delta.$$

As a result, the total number of projects that are screened out in this stage is

$$\int_0^{\delta} p(a, \alpha) d\alpha.^4$$

An example of such a screening technology is as follows. Each project is reviewed with probability  $a$ , which is the planner's effort level and is normalized to be in the interval  $[0,1]$ . When a project is reviewed, its true type will be found. No new information is acquired from projects that are not reviewed. Therefore, each project sends one of the three mutually exclusive and collectively exhaustive signals: (1) it is an *ex ante* efficient project, i.e.  $\alpha \geq \delta$ ; (2) it is an *ex ante* inefficient project, i.e.  $\alpha < \delta$ ; (3) no new information is acquired, so that the expected profit of the project is  $E\{\alpha\} - c - i$ . We assume that the prior expected profit is non-negative, i.e.  $E\{\alpha\} - c - i \geq 0$ . Since there are a continuum of projects, by the law of large numbers, the planner should fund the project if and only if its expected profit is non-negative. Therefore, a project should be funded if and only if signal (1) or (3) is observed.<sup>5</sup> In this example,

$$p(a, \alpha) = \begin{cases} 0 & \text{if } \alpha \geq \delta; \\ a & \text{if } \alpha < \delta. \end{cases}$$

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<sup>4</sup>Under the assumption that the planner funds a project if and only if it passes the screening, the first-best can be achieved if the principal can observe the number of funded projects. However, the above assumption is not necessarily true; the planner can choose not to fund some projects that pass the screening. For the sake of simplicity, we maintain the above assumption and assume furthermore that the principal cannot observe the number of funded projects. The main results of the paper remain to hold if we abandon both of the above assumptions.

<sup>5</sup>Such a screening technology is very similar to that used by many to study the monitoring problem in hierarchies, e.g., Calvo and Wellisz (1978). In the monitoring problem, due to the supervisor's time constraint, the probability that a subordinate's effort level is checked is smaller than 1. If not checked, a subordinate is assumed to have made the prespecified effort and paid a wage accordingly. If checked, the subordinate's real effort level is perfectly revealed. The subordinate is then paid the prespecified wage or suffer a penalty depending on the observed effort level. Qian (1994) also uses such a screening technology.

At time 1, the types of the projects that are funded in the first stage are revealed to the planner, who then chooses to terminate the worst  $\hat{t}$  of the remaining

$$1 - \int_0^{\delta} p(a, \alpha) d\alpha$$

bad projects. The types of the projects are not revealed to the principal.

In the second stage, an additional investment of  $\$i$  is required on each project that is not terminated at time 1 to realize its return. Since the first-stage investment of  $\$c$  is sunk, a project should be terminated if and only if  $r(\alpha) < i$  and the *ex post* efficient number of projects to be terminated,  $\hat{t}'$ , is

$$\hat{t}' = \int_0^{\xi} [1 - p(a, \alpha)] d\alpha,$$

where  $r(\xi) = i$  (see Figure 1).

At time 2, the returns of the retained projects are realized and the planner is rewarded according to the realized gross value-added and the number of terminations made at time 1.

We assume that the gross value-added is

$$x = y(a, \hat{t}) + \theta.$$

$\hat{t}$  is the number of projects terminated by the planner at time 1.  $y(a, \hat{t})$  is the expected gross value-added. When

$$\int_0^{\delta} [1 - p(a, \alpha)] d\alpha \geq \hat{t},$$

that is, the number of projects to be terminated is no greater than the number of *ex ante* inefficient projects that survive the initial screening,

$$y(a, \hat{t}) = \int_{\delta}^1 [r(\alpha) - i] d\alpha + \int_{\hat{\alpha}(a, \hat{t})}^{\delta} [r(\alpha) - i] [1 - p(a, \alpha)] d\alpha - [1 - \int_0^{\delta} p(a, \alpha) d\alpha] c, \quad (1)$$

where,  $\hat{\alpha}(a, \hat{t})$  is the worst project that survives the time 1 elimination and is defined by

$$\int_0^{\hat{\alpha}(a, \hat{t})} [1 - p(a, \alpha)] d\alpha = \hat{t}. \quad (2)$$

The first term of  $y(a, \hat{t})$  is the expected total *ex post* profits from projects with revenue  $r(\alpha) > c + i$ . The second term is the expected total *ex post* profits from the continued projects with revenue  $r(\alpha) < c + i$ . *Ex post* profits are computed without accounting for the costs of first-stage investments. The third term is the total costs of first-stage investments of all funded projects.  $\theta$  denotes the unexpected part of the gross value-added and is a random variable with mean 0 and probability density function  $g(\theta)$ . The realization of  $\theta$  is not observable by the principal.<sup>6</sup> The planner's reward is based on the values of  $x$  and  $\hat{t}$ .

Assume that planner's utility function is

$$v(w) - d(a),$$

where  $w$  is the income of the planner and  $a$  is his effort. Assume that  $v' > 0$  and  $v'' < 0$ . If the principal pays  $w(x, \hat{t})$  to the planner, the expected utility of the planner is

$$u(a, \hat{t}) = E_{\theta} \{v[w(x(a, \hat{t}, \theta), \hat{t})] - d(a)\}$$

when he exerts an effort  $a$  in the first stage and terminates  $\hat{t}$  projects in the second stage.

The principal's objective function is

$$\pi = E_{\theta} \{x(a, \hat{t}, \theta) - w(x(a, \hat{t}, \theta), \hat{t})\}.$$

Our assumption that the types of the projects are not revealed to the principal after the initial investment is very important. Otherwise, the principal would be able

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<sup>6</sup>  $\theta$  is a common shock. Without this common shock, the first-best effort of the planner can be implemented by "selling the economy to the planner", i.e. demanding a certain payment from the planner and giving him all the residual value-added. When there is no uncertainty, this arrangement is efficient even when the planner is risk averse or/and has limited liability. Idiosyncratic shocks are not sufficient, because the number of firms is large and, by the law of large numbers, idiosyncratic shocks will be averaged away.

to induce the first-best screening effort and *ex post* efficient termination decision from the planner by using an incentive scheme that is contingent on the observed types of the projects and the value-added, which is the case if the securities of each project is publicly traded on the capital market.<sup>7</sup> This is why the soft budget constraint problem is much less severe in capitalist economies, where the principal is the investors. Proposition 1 states a stronger result: in order to induce the first-best screening effort, the principal only needs to know the number of *ex ante* (or *ex post*) inefficient projects that survive the time 0 screening.

Proposition 1: If the principal knows the number of *ex ante* (or *ex post*) inefficient projects that survives the time 0 screening, then she will know the agent's effort in screening the projects and therefore can induce the first-best screening effort and *ex post* efficient termination decision from the agent.

Proof: The number of *ex ante* inefficient projects that survive the screening is

$$t = \int_0^{\delta} [1 - p(a, \alpha)] d\alpha.$$

By the assumption that  $\frac{\partial p}{\partial a} > 0$  for  $\alpha < \delta$ ,  $t$  is a strictly decreasing function of  $a$ .

Therefore the principal can infer  $a$  from her observation of  $t$ . Similarly, the principal can infer  $a$  from the number of *ex post* inefficient projects that survives the screening,  $\hat{t}' = \int_0^{\xi} [1 - p(a, \alpha)] d\alpha$ . When the principal can infer the screening

effort  $a$ , she can mandate the agent to exert the first-best effort at time 0 and to close *ex post* efficient number of projects at time 1. Q.E.D.

### 3. The Soft Budget Constraint

In this section, we show that, fewer projects are terminated at time 1 than the *ex post* efficient number. First, let's discuss the form of the optimal contracts.

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<sup>7</sup>We assume that Efficient Market Hypothesis holds for the capital market. Later we will discuss the comparison between the capitalist economy and the socialist economy when Efficient Market Hypothesis does not hold.

The general form of the contract is for the principal to offer a wage function  $w(x, \hat{t})$ . Given this wage function, the agent chooses his effort,  $a$ , at time 0 and the number of projects to terminate,  $\hat{t}$ , at time 1 to maximize his expected utility. Since  $\hat{t}$  is verifiable, the principal can mandate  $\hat{t} = \tilde{t}$ , for some  $\tilde{t}$ , in a contract together with a wage function  $w(x)$ . Doing so is equivalent to offering a wage contract of the following special form:

$$w(x, \hat{t}) = \begin{cases} w(x) & \text{if } \hat{t} = \tilde{t}; \\ -\infty & \text{otherwise.} \end{cases} \quad (\text{SF})$$

Given such a contract, the agent chooses the effort level  $a$  to maximize his utility.

Proposition 2: Restricting the wage contracts to the special form given in (SF) does not reduce the principal's optimal payoff; that is, the optimal contract can be chosen to be of the special form.

Proof: Suppose the optimal wage contract is  $w^*(x, \hat{t})$ , and under this contract,  $a^*$  and  $\hat{t}^*$  are induced. If the principal mandates  $\hat{t}^*$  and offers a wage contract

$$\hat{w}(x) = w^*(x, \hat{t}^*),$$

then the agent chooses  $a$  to maximize

$$\max_a u(a, \hat{t}^*) = \max_a E_\theta \{v[w^*(x(a, \hat{t}^*), \theta), \hat{t}^*] - d(a)\}$$

By the definition of  $a^*$  and  $\hat{t}^*$ , the agent's optimal effort is  $a^*$ . That is, mandating  $\hat{t}^*$  and offering  $\hat{w}(x)$  induce the same effort as offering  $w^*(x, \hat{t})$ . The principal gets

$$\pi = E_\theta \{x(a^*, \hat{t}^*, \theta) - w(x(a^*, \hat{t}^*, \theta), \hat{t}^*)\},$$

which is the net revenue that the principal gets under the optimal contract  $w^*(x, \hat{t})$ .

That is, the maximum net revenue that the principal can attain by choosing the optimal contract from all possible contracts is attained by choosing one,

$$w(x, \hat{t}) = \begin{cases} \hat{w}(x) & \text{if } \hat{t} = \hat{t}^*; \\ -\infty & \text{otherwise,} \end{cases}$$

from the special class of contracts.

Q.E.D.

Now we can start to solve the optimization problem. Since  $x = y(a, \hat{t}) + \theta$  and the probability density function of the random variable  $\theta$  is  $g(\theta)$ , the probability density function of  $x$  is

$$g(x - y(a, \hat{t})) = f(x; a, \hat{t}). \quad (3)$$

Given  $\hat{t}$  and  $w(x)$ , the agent solves

$$\max_a u(a, \hat{t}) = \max_a \int v(w(x)) f(x; a, \hat{t}) dx - d(a).$$

The first-order condition of the problem is

$$\int v(w(x)) f_a(x; a, \hat{t}) dx - d'(a) = 0.$$

The agent's second-order condition is

$$\int v(w(x)) f_{aa}(x; a, \hat{t}) dx - d''(a) \leq 0.$$

In this paper, we make the following technical assumption:

$$(SOC-A) \quad \int v(w(x)) f_{aa}(x; a, \hat{t}) dx - d''(a) < 0.$$

The principal solves

$$\begin{aligned} & \max_{\hat{t}, a, w(x)} \int [x - w(x)] f(x; a, \hat{t}) dx \\ & \text{s.t. } \int v(w(x)) f_a(x; a, \hat{t}) dx - d'(a) = 0 \quad (\text{IC}) (\mu) \\ & \text{and } \int v(w(x)) f(x; a, \hat{t}) dx - d(a) \geq 0 \quad (\text{IR}) (\lambda) \end{aligned} \quad (4)$$

The first constraint is the incentive compatibility constraint and says that the effort induced is determined by the agent's first-order condition.<sup>8</sup> The second constraint is the individual rationality constraint and says that the contract has to at least provide the agent his reservation utility, which is assumed to be 0 here.

The principal's optimization problem (4) can be reformulated as follows:

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<sup>8</sup>We assume that the maximum likelihood ratio condition holds and the distribution function of  $x$  is convex in  $a$  so that the first-order condition approach works here. (See Rogerson (1985))

$$\max_{\hat{t}} \Pi(\hat{t})$$

where,

$$\begin{aligned} \Pi(\hat{t}) &= \max_{w(x), a} \int [x - w(x)] f(x; a, \hat{t}) dx \\ \text{s. t. } & \int v(w(x)) f_a(x; a, \hat{t}) dx - d'(a) = 0 \quad \text{(IC) } (\mu) \\ & \text{and } \int v(w(x)) f(x; a, \hat{t}) dx - d(a) \geq 0 \quad \text{(IR) } (\lambda) \end{aligned} \quad (5)$$

Let's continue to use  $\hat{t}^*$  to denote the *ex ante* optimal number of projects for the planner to terminate at time 1 mandated in the contract by the principal. The first-order condition for  $\hat{t}^*$  is

$$\Pi'(\hat{t}^*) = 0. \quad (6)$$

The second-order necessary condition for  $\hat{t}^*$  to be the maximum is  $\Pi''(\hat{t}^*) \leq 0$ . We make the following technical assumption:

$$\text{(SOC-P)} \quad \Pi''(\hat{t}) < 0 \text{ for all } \hat{t}.$$

The Lagrangian of program (5) is

$$\begin{aligned} L &= \int [x - w(x)] f(x; a, \hat{t}) dx + \mu \left\{ \int v(w(x)) f_a(x; a, \hat{t}) dx - d'(a) \right\} \\ & \quad + \lambda \left\{ \int v(w(x)) f(x; a, \hat{t}) dx - d(a) \right\}. \end{aligned}$$

Pointwise optimization of the Lagrangian with respect to the sharing rule,  $w(x)$ , and rearranging yield

$$\frac{1}{v'(w(x))} = \lambda + \mu \frac{f_a(x; a, \hat{t})}{f(x; a, \hat{t})} \quad \text{(FOC-w)} \quad (7)$$

By equation (3), the definition of  $f(x; a, \hat{t})$ ,

$$f_a(x; a, \hat{t}) = -g'(x - y(a, \hat{t})) \frac{\partial y}{\partial a}.$$

Therefore,

$$\frac{f_a(x; a, \hat{t})}{f(x; a, \hat{t})} = - \frac{g'(x - y(a, \hat{t}))}{g(x - y(a, \hat{t}))} \frac{\partial y}{\partial a}.$$

If  $\mu > 0$ ,  $\frac{\partial y}{\partial a} > 0$ , and  $\frac{g'(x)}{g(x)}$  is a decreasing function of  $x$ , then the right hand side of

(FOC-w) is an increasing function of  $x$ . By (FOC-w),  $w$  is an increasing function of  $x$  given the above conditions, because  $\frac{1}{v'(w)}$  is an increasing function of  $w$ .

Lemma 1:  $w(x)$  is an increasing function if  $\mu > 0$ ,  $\frac{\partial y}{\partial a} > 0$ , and  $\frac{g'(x)}{g(x)}$  is a decreasing

function of  $x$ .

$\mu$  is the multiplier of the constraint (IC) in program (5) and  $\mu > 0$  means that the principal would like to see a higher effort from the agent. The other two conditions in the proposition are simply another form of the monotone likelihood ratio

condition that is often used in the principal-agent literature. The condition that  $\frac{\partial y}{\partial a} > 0$  means that the expected gross value-added is an increasing function of the

planner's effort in screening the projects. The condition that  $\frac{g'(x)}{g(x)}$  is a decreasing

function of  $x$  is satisfied by the normal distribution and the t-distribution, among others, and implies that

$$\frac{g'(x - y(a, \hat{t}))}{g(x - y(a, \hat{t}))}$$

is an increasing function of  $y(a, \hat{t})$ , which in turn implies that higher values of  $x$  signal higher values of  $y(a, \hat{t})$ . The two conditions together imply that higher values of  $x$  signal higher values of  $a$ . It is natural that under these conditions, the agent should be paid more when the value of  $x$  is higher. In the rest of this paper, we assume the monotone likelihood ratio condition:

(MLR)  $\frac{g'(x)}{g(x)}$  is a decreasing function of  $x$ .

Lemma 2: The Lagrangian multiplier,  $\mu$ , of the incentive compatibility constraint, (IC), is positive if  $\frac{\partial y}{\partial a} > 0$ .

Proof: See the appendix.

This lemma says that the principal would like to see a higher effort from the agent if  $\frac{\partial y}{\partial a} > 0$ , which is a natural result.

Proposition 3: Assume (MLR). Then given any  $\hat{t}$ , at the optimal solution to program (5),  $\frac{\partial y}{\partial a} > 0$ . Consequently,  $w(x)$  is an increasing function and  $\mu > 0$ .

Proof: We prove the proposition by contradiction. Assume that  $\frac{\partial y}{\partial a} \leq 0$ . We derive contradictions for two cases:

Case 1:  $\mu \leq 0$ .

In this case, (FOC-w) implies that  $w(x)$  is a non-decreasing function. Then,

$$\begin{aligned} & \int v(w(x))f_a(x; a, \hat{t})dx - d'(a) \\ &= -\frac{\partial y}{\partial a} \int v(w(x))g'(x; a, \hat{t})dx - d'(a) \\ &= \frac{\partial y}{\partial a} \int v'(w(x))w'(x)g(x; a, \hat{t})dx - d'(a) < 0. \end{aligned}$$

The first of the above equations is due to the definition of function  $f$ . The second equation is derived by integrating by parts. The last inequality contradicts the incentive compatibility constraint, (IC).

Case 2:  $\mu > 0$ .

In this case, (FOC-w) implies that  $w(x)$  is a non-increasing function. Then,

$$\begin{aligned} & \int [x - w(x)]f_a(x; a, \hat{t})dx \\ &= -\frac{\partial y}{\partial a} \int [x - w(x)]g'(x; a, \hat{t})dx \\ &= \frac{\partial y}{\partial a} \int [1 - w'(x)]g(x; a, \hat{t})dx \leq 0. \end{aligned}$$

Again, the first of the above equations is due to the definition of function  $f$ . The second equation is derived by integrating by parts. The last inequality contradicts either (FOC-a) or (SOC-A). It follows from Lemmas 1 and 2 that  $w(x)$  is an increasing function and  $\mu > 0$ . Q.E.D.

The intuition of this proposition is very clear. In the first case, the principal would like to see a lower effort and effort reduces the output. Therefore the

principal chooses the reward to the agent as increasing in the output. Given this incentive scheme, the agent would also like to reduce his effort. This cannot be true at the second-best optimum. In the second case, the principal would like to see a higher effort, since  $\mu > 0$ . Because of this and because effort reduces the output, the principal chooses the reward to the agent as decreasing in the output. Then the principal's net revenue,  $x - w(x)$ , increases in the output,  $x$ . It in turn implies that the principal would like to see a lower effort since it leads to more output, which contradicts with  $\mu > 0$ .

To prepare for the main result, we first study some properties of  $y(a, \hat{t})$ .

Lemma 3: When  $\hat{\alpha}(a, \hat{t}) \leq \delta$ ,  $\frac{\partial y}{\partial \hat{t}}(a, \hat{t}) = -r[\hat{\alpha}(a, \hat{t})] + i$ .

Proof: See the appendix.

Lemma 4: When  $\hat{\alpha}(a, \hat{t}) \leq \delta$ ,  $\frac{\partial y}{\partial a} > 0$ , and  $\frac{\partial^2 y}{\partial \hat{t} \partial a} < 0$ .

Proof: See the appendix.

Let's use  $\hat{t}'$  to denote the *ex post* efficient number of terminated projects.  $\hat{t}'$  maximizes the expected gross value-added  $y(a, \hat{t})$  and satisfies

$$\frac{\partial y}{\partial \hat{t}}(a, \hat{t}') = -r[\hat{\alpha}(a, \hat{t}')] + i = 0,$$

or  $\hat{\alpha}(a, \hat{t}') = \xi$ . The planner would choose  $\hat{t} = \hat{t}'$  if he was not mandated otherwise and his payoff did not depend on the number of terminated projects, as is the case in Dewatripont and Maskin (1990). However, in order to induce the planner's effort in screening the projects, the principal orders the planner to shut down fewer projects than  $\hat{t}'$ , as we will prove in proposition 4.

Proposition 4: The number of projects terminated in the second stage under the *ex ante* optimal contract,  $\hat{t}^*$ , is less than the *ex post* efficient number of terminated projects,  $\hat{t}'$ .

Proof: By the Envelop Theorem, since  $\Pi(\hat{t})$  is the value of the net revenue when  $w(x)$  and  $a$  are chosen optimally, given  $\hat{t}$  and subject to (IR) and (IC) constraints,

$$\Pi'(\hat{t}) = \frac{\partial L}{\partial \hat{t}}(\hat{t}; w_{\hat{t}}(x), a(\hat{t})),$$

where,  $L$  is the Lagrangian of program (5) and

$$\frac{\partial L}{\partial \hat{t}} = \int [x - w(x)] f_{\hat{t}}(x; a, \hat{t}) dx + \mu \int v(w(x)) f_{a\hat{t}}(x; a, \hat{t}) dx + \lambda \int v(w(x)) f_{\hat{t}}(x; a, \hat{t}) dx,$$

and  $w_{\hat{t}}(x)$  is the optimal wage function given  $\hat{t}$ , and  $a = a(\hat{t})$  is the agent's optimal effort given  $w_{\hat{t}}(x)$  and  $\hat{t}$ . The first-order condition with respect to  $\hat{t}$  is

$$\Pi'(\hat{t}^*) = 0. \quad (6)$$

Because  $f(x; a, \hat{t}) = g(x - y(a, \hat{t}))$ ,

$$f_{\hat{t}}(x; a, \hat{t}) = -g'(x - y(a, \hat{t})) \frac{\partial y}{\partial \hat{t}}(a, \hat{t}),$$

and

$$f_{a\hat{t}}(x; a, \hat{t}) = g'' \frac{\partial y}{\partial \hat{t}}(a, \hat{t}) \frac{\partial}{\partial a}(y(a, \hat{t})) - g' \frac{\partial}{\partial a} \left( \frac{\partial y}{\partial \hat{t}}(a, \hat{t}) \right).$$

By the definition of  $\hat{t}'$ ,

$$\frac{\partial y}{\partial \hat{t}}(a, \hat{t}') = 0.$$

Substituting the last three equations into  $\frac{\partial L}{\partial \hat{t}}$  yields

$$\left. \frac{\partial L}{\partial \hat{t}} \right|_{\hat{t}=\hat{t}'} = -\mu \frac{\partial^2 y}{\partial \hat{t} \partial a}(a, \hat{t}') \int v(w(x)) g'(x - y(a, \hat{t}')) dx.$$

Integration by parts yields

$$\left. \frac{\partial L}{\partial \hat{t}} \right|_{\hat{t}=\hat{t}'} = \mu \frac{\partial^2 y}{\partial \hat{t} \partial a}(a, \hat{t}') \int v'(w(x)) w'(x) g(x - y(a, \hat{t}')) dx,$$

because  $g$  is a probability density function. By Proposition 3,  $w'_r(x) > 0$ .

Therefore,

$$\begin{aligned}\Pi'(\hat{t}') &= \frac{\partial L}{\partial \hat{t}}(\hat{t}'; w_{\hat{t}'}(x), a(\hat{t}')) \\ &= \mu \frac{\partial^2 y}{\partial \hat{t} \partial a}(a, \hat{t}') \int v'(w_{\hat{t}'}(x)) w'_{\hat{t}'}(x) g(x - y(a(\hat{t}'), \hat{t}')) dx < 0,\end{aligned}$$

since by Lemma 4,

$$\frac{\partial^2 y}{\partial \hat{t} \partial a}(a, \hat{t}') < 0.$$

Combining this inequality with (6) yields

$$\Pi'(\hat{t}') < 0 = \Pi'(\hat{t}^*). \quad (7)$$

This inequality implies that the *ex post* optimal  $\hat{t}$  is not *ex ante* optimal; in other words, at the *ex ante* optimum, the number of bad projects to be shut down is not *ex post* efficient. By Assumption (SOC-P), inequality (7) implies that

$$\hat{t}^* < \hat{t}';$$

that is, the *ex ante* optimal number of firm to be shut down is smaller than the *ex post* efficient number. This result is exactly what we wanted: too few projects are shut down *ex post*, due to the above agency problem. Q.E.D

From the proof, we can see that the principal would choose  $\hat{t}$  if the effect of  $\hat{t}$  on the effort was not considered; i.e. if  $\mu = 0$ . However, limiting the number of terminations makes it impossible for the planner to close down as many inefficient projects as he wants and it in turn raises the cost of funding too many inefficient projects in stage 1 due to low effort; lowering  $\hat{t}$  increases the marginal expected revenue of the effort,  $\frac{\partial y}{\partial a}(a, \hat{t})$ , and consequently increases the marginal utility of the effort to the planner. The principal then will choose  $\hat{t}$  to be smaller than  $\hat{t}'$  because it makes it easier to motivate the planner.

Remark 1: The only property of the function  $y(a, \hat{t})$  that we used is

$$\frac{\partial^2 y}{\partial \hat{t} \partial a}(a, \hat{t}') < 0 \text{ at } \hat{t}' \text{ where } \frac{\partial y}{\partial \hat{t}}(a, \hat{t}') = 0.$$

Therefore, all the results hold for a general model where an agent chooses an unobservable action  $a$  and an observable action  $\hat{t}$ , the output is  $x = y(a, \hat{t}) + \theta$ , and the principal chooses an incentive scheme to maximize expected profit, as long as the function  $y(a, \hat{t})$  satisfies the above property.

Remark 2: The optimal contract can also be represented as

$$w(x, \hat{t}) = \begin{cases} \hat{w}(x) & \text{if } \hat{t} \leq \hat{t}^*; \\ -\infty & \text{otherwise.} \end{cases}$$

This incentive scheme can be interpreted as follows. If the number of shut-down projects is not higher than a specified threshold, the planner will be rewarded according to the gross value-added. If the number exceeds the threshold, the principal will interpret it as the result of low effort and will punish the planner severely.

Remark 3: The reason we have such a discontinuous incentive scheme with an infinite penalty in some cases is because there is no uncertainty about the number of shut-down projects. If the noise about the number of shut-down projects has a probability distribution with support from  $-\infty$  to  $\infty$ , the incentive contract for the risk averse planner will not involve infinite penalty. In such a case, the reward function to the planner,  $w(x, \hat{t})$ , should be smooth and decreasing in  $\hat{t}$  because a higher  $\hat{t}$  indicates lower effort. As a result, the planner will terminate fewer than *ex post* efficient number of projects at time 1.

#### 4. Renegotiation

We have shown that when there is asymmetric information about the planner's effort and also the type of projects that survived the initial screening, the state's *ex ante* optimal incentive requires that the planner close fewer projects than what is *ex post* optimal. However, one can argue that, after the planner has made the effort, a Pareto efficient new contract can be signed through renegotiation because the

original contract leads to *ex post* inefficient project termination decisions and does not provide the planner full insurance. Presumably, a new contract that specifies a fixed wage for the agent would induce the planner to close all *ex post* inefficient projects and also fully insure the planner against any risk associated with output fluctuations.<sup>9</sup> A legitimate question is whether or not the main result of the model, that the planner does not to close all *ex post* inefficient projects under the *ex ante* optimal incentive scheme, is robust when renegotiation is allowed?

In this section, we show that the result of our model does not change if the renegotiation follows the process specified by Ma (1994).

To facilitate the discussion, we reorganize the principal's optimization problem along the line of Grossman and Hart (1983). Given an effort level  $a$ , there is a compensation scheme  $w(x, \hat{t}; a)$  that induces the agent to choose the effort level  $a$  at least cost to the principal. We call  $w(x, \hat{t}; a)$  the incentive-efficient scheme for the effort level  $a$ , which is the solution to the following optimization program :

$$\begin{aligned} & \min_{w(x, \hat{t})} \int w(x, \hat{t}) f(x; a, \hat{t}) dx \\ \text{s.t. } & \max_{\hat{t}} \int v(w(x, \hat{t})) f(x; a, \hat{t}) dx - d(a) \geq \\ & \max_{\hat{t}} \int v(w(x, \hat{t})) f(x; a', \hat{t}) dx - d(a') \text{ for all } a' \text{ (IC)} \\ & \text{and } \max_{\hat{t}} \int v(w(x, \hat{t})) f(x; a, \hat{t}) dx - d(a) \geq 0 \quad \text{(IR)} \end{aligned}$$

Note that constraint (IR) in the program is binding. The principal then chooses to implement an effort level using the corresponding incentive-efficient scheme to maximize her expected payoff.

An allocation is defined as a collection of an effort level, a number of projects that are terminated, and a compensation scheme. The second-best allocation under full commitment is  $[a^*, \hat{t}^*, w^*(x, \hat{t})]$ .

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<sup>9</sup>We assume that when the planner is indifferent between different actions, he takes the action that is best for the principal. Alternatively, the payoff to the planner should increase slightly with the total surplus.

In this section, we assign a different meaning to the term *contract* from that in previous sections. Here, a contract is defined as a menu of many compensation schemes. For example, the collection of incentive-efficient schemes,  $C_0 = \{w(x, \hat{t}; a)\}_{a \geq 0}$ , is a contract. We call it the incentive-efficient contract.

The timing of the renegotiation game is as follows.

The principal proposes a contract $C$ .	The principal chooses between $C$ and $D$ .
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The agent exerts effort $a$ and proposes another contract $D$ .	The agent chooses $\hat{t}$ and a scheme from the contract chosen by the principal.
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Given a contract  $C$ , an action that the principal takes in the first stage, there is a subgame that starts with it. We identify the subgame with the contract  $C$ . Following Ma (1994) and adapting to the introduction of an additional choice variable, the number of projects to close, we can prove the following proposition.

Proposition 5: The second-best allocation under full commitment,  $[a^*, \hat{t}^*, w^*(x, \hat{t})]$ , can be implemented by a perfect-Bayesian equilibrium of the subgame  $C_0$  in the above renegotiation game.

The proposition says that the incentive scheme that leads to soft budget is the result of a perfect-Bayesian equilibrium of the renegotiation game. The proof of Proposition 5 is offered in the appendix. Ma also proves the uniqueness of the equilibrium, subject to a refinement imposed on the principal's beliefs, in the simple principal-agent context. The proof for uniqueness in our context is beyond the scope of this paper.

The intuition for the result is simple. “[A]t renegotiation the principal’s belief can depend on the agent’s new contract offer. For example, suppose the principal initially proposes an incentive contract. Then the principal can believe that the agent has taken an inferior action if she receives a full insurance new proposal

from the agent, and she may indeed reject it. Anticipating a rejection of his renegotiation offer, the agent chooses a costly action.” (Ma, 1994, p.110)

Having proved that the result of the model is robust in the renegotiation game that follows the process specified by Ma (1994), we note that the equilibrium of a renegotiation game is in general very sensitive to the specification of the renegotiation process. Thus the result in Appendix 2 must be considered tentative before other specifications of the renegotiation problem are explored.<sup>10</sup>

It can also be noted that there are reasonable ways to extend the model so that the *ex ante* optimal contract is renegotiation-proof. One familiar approach is to consider the reputation effect. The principal’s concern with her reputation can provide an incentive for her to refrain from renegotiating the wage contract with the agent. Formally, an infinitely repeated game between the same principal and generations of different agents may be modeled. When the principal's discount factor is sufficiently large, in equilibrium she will not renegotiate with any agent.<sup>11</sup>

Introducing the learning problem similar to that of Holmstrom and Ricart i Costa (1986) to our model would be an interesting extension in its own sake and also give a reason for the contract to be renegotiation proof. In a learning model in which the game would be repeated twice, the agent's ability would be initially unknown but revealed by the number of projects closed in the first period. If closing more projects signals the planner's low ability and thereby reduces his

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<sup>10</sup> Although a full exploration of all possible renegotiation games is beyond the scope of this paper, it can be noted that, in Fudenberg and Tirole (1990) where the principal proposes a new contract after the agent has taken his action, the set of implementable actions is much smaller. Also a fixed wage offer by the principal as found in Hermalin and Katz (1991) is not feasible in our model because, unlike in their model in which the principal receives a signal of the agent's action, there is no such a signal in our model and the asymmetric information between the planner and the state persists through the first stage. Without information about the planner's effort, bad projects passed the initial screening and thus also the expected payoff of the planner, the state cannot offer a utility-equivalent fixed wage.

<sup>11</sup>In Dewatripont and Maskin (1990), renegotiation is not prevented even in a repeated game. The reason is that, unlike the number of closed projects in our model, the profitability of a project in their model is not public information.

second period wage (assuming that the state cannot commit to a wage for the second period independent of the planner's ability), the planner would close fewer projects in the first period than what is socially optimal to avert the unfavorable signal that has a detrimental consequence for him in the second period. If the game is repeated for any finite number of times, the planner will close fewer than socially optimal number of projects in every period except the last one.

## 5. Summary and Concluding Remarks

We have shown that the state's *ex ante* optimal incentive requires the agent to close fewer projects than what is *ex post* optimal, giving rise to the soft-budget problem observed in socialist economies. The rationale for this somewhat counter intuitive result can be understood by noticing the following critical elements of the model. First, the agent's wage is tied to the final output. Second, the expected value of the final output is determined by two successive actions of the agent: effort to screen projects before the initial investment and decision on the number of projects to close after the initial but before the second investment. Finally, the effect of effort on the expected output and thus the planner's expected wage depend on the number of *ex post* inefficient projects that the planner can close. If an *ex post* inefficient project survives the screening at Time 0 but is closed at Time 1, the expected cost to the planner is the wage loss corresponding to the output loss of  $\$c$ . If an *ex post* inefficient project survives the screening and is later refinanced, the expected cost to the planner is a greater wage loss corresponding to the output loss of  $\$(c+i-r(\alpha)) > \$c$ . This means that the fewer *ex post* inefficient projects to be closed, the higher is the marginal return to the agent's *ex ante* effort, and the harder the planner will work *ex ante* to screen out inefficient projects. Since at  $\xi$  the marginal cost of closing one fewer project is almost zero whereas the marginal gain of additional effort is greater than zero, the tradeoff is worthwhile and the optimal

incentive calls for not closing all *ex post* inefficient projects, leading to the observed soft-budget problem.

The key information assumption that led to the result is that after the initial investment, the only thing the principal can observe besides the output is the number of projects closed by the planner. If the principal has perfect information about the agent's effort, e.g., if she knows exactly how many bad projects survived the initial screening, then first-best allocation can be achieved, as is shown in Proposition 1. In the less extreme case where the principal gets some signal about the agent's effort in addition to the output and the number of closed projects, the soft budget problem still exists but is less severe.<sup>12</sup>

In the theory of the firm, unified ownership is generally viewed as the solution to the holdup problem in market transactions involving relation-specific investment. Williamson (1985), however, raised the question about the cost of the unified governance inside the firm. Efforts have been made to understand the costs of replacing market with unified organizational governance. (See Holmstrom and Tirole, 1988, for a concise review and discussion.) Our model contributes to the literature by showing that an informational condition for the soft budget problem to arise is the principle's dependence on the agent for information about the agent's effort and project performance. Since in capitalist firms it is also common that principals have to rely upon agents for this kind of information, the model predicts the soft-budget problem inside capitalist firms as well. There seem to be ample (descriptive) empirical evidence supporting the prediction. For example, supervisors who recruited and has the responsibility to motivate the subordinates to work hard are often found unwilling to rate the subordinates low in performance

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<sup>12</sup>Holmstrom (1979) shows that an informative signal about the effort can always improve the efficiency of the optimal principal-agent contract. Furthermore, if we use second-order stochastic dominance to rank the accuracy of signals, then the result about sufficient statistics in Holmstrom (1982) implies that more accurate signal leads to more efficient outcome.

appraisals.<sup>13</sup> As Milgrom and Roberts (1993, p.406) suggest, one reason for this is that poor performance rating of the subordinates reflects badly on the supervisor himself. Daft (1995, p.390), after giving examples of managers throwing good money after the bad, wrote: "Research suggests that organizations often continue to invest time and money in a solution despite strong evidence that it is not working." And an explanation is that managers tend to "block or distort negative information when they are personally responsible for a negative decision."

The difference between capitalist and planned socialist economies, however, is found at the boundary of the firm. In the former, the securities of a firm are traded in the market, revealing to the public the information about project qualities from which the principal can infer the agent's effort before the return of a project is realized. The informational reason for the soft-budget problem is thus undermined at the boundary of the firm in capitalist market economies.<sup>14</sup> The assumption of Proposition 1 is satisfied if the principal is the investors, the securities of the projects are traded on the capital market and Efficient Market Hypothesis holds. Even if Efficient Market Hypothesis does not hold, the prices of the securities of the projects still contain some information about the types of the projects, hence about the effort of the agent. Incorporating such information by including the security prices in the incentive contract will help motivating the agent and mitigating the soft budget constraint problem.<sup>15</sup>

The above discussion has emphasized the importance of an equity market in generating signals on project performance that can be used to infer the *ex ante* effort of the agent and the critical role of these signals in hardening the budget

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<sup>13</sup>See also Milgrom and Roberts (1993, p.404-406) for some cited evidence.

<sup>14</sup>The kind of socialism we modeled here is the classic type characterized by central planning. Whether market socialism can do away with the problem by introducing market discipline to firms is an issue that we don't deal with here. See Bardhan and Roemer (1993) and Shleifer and Vishny (1994) for opposing views on the issue.

<sup>15</sup>See footnote 12.

constraint. Centrally planned socialist economies (or classic socialism, to use Kornai's phrase) did not have such a market and thus in our opinion faced insurmountable informational difficulties in overcoming the pervasive soft budget problem. Year after year, the Politbureau of the Party in the former Soviet Union and other countries of classic socialism readily agreed with the planners or other agents that poor performance of projects is excusable because of "bad weather," not because of the lack of effort. Recent discussions of market socialism has seen an increased recognition of the importance of the equity market with some fairly detailed proposals as how such a market might be organized. (See Bardhan and Roemer, 1993.) To what extent market socialism can mimic the equity market in capitalist market economies in hardening the budget constraint, however, remains an open question. This paper does not directly address this question and interested readers are referred to Bardhan and Roemer (1992) and Shleifer and Vishny (1994) for opposing views. However, the result of this paper does suggest that, to understand the efficiency of market socialism relative to that of a capitalist market economy, it is important to understand how the equity markets under the two systems generate information on manager's effort and the performance of individual firms (projects). In socialist market economies, the state is the dominant shareholder, while in capitalist market economies, shares tend to be more diffusely held. In the later case, on the one hand, the free rider problem discourages small shareholders to monitor the management (Shleifer and Vishny (1986)); on the other hand, the existence of liquidity traders gives speculators incentives to collect information (Holmstrom and Tirole (1993)). In the former case, the state can use internal auditing to obtain information. However, this creates an agency problem with the auditor. Investigating this issues in the context of this model will be the topic of another major research work.

## Appendix

The proof of Lemma 2:<sup>16</sup>

This is a proof by contradiction. Suppose  $\mu \leq 0$ . Define  $\bar{w}$  such that

$$\lambda = \frac{1}{v'(\bar{w})}.$$

By (FOC-w),  $w(x) \leq \bar{w}$  when  $f_a > 0$  and  $w(x) \geq \bar{w}$  when  $f_a \leq 0$ . Therefore,

$$\begin{aligned} \int [x - w(x)] f_a dx &\geq \int (x - \bar{w}) f_a dx \\ &= \frac{\partial}{\partial a} \int (x - \bar{w}) f(x; a, \hat{t}) dx \\ &= \frac{\partial}{\partial a} [y(a, \hat{t}) - \bar{w}] = \frac{\partial y}{\partial a} > 0. \end{aligned}$$

This contradicts with first-order condition with respect to  $a$  of the principal's optimization problem,

$$\frac{\partial L}{\partial a} = \int [x - w(x)] f_a(x; a, \hat{t}) dx + \mu \left\{ \int v(w(x)) f_{aa}(x; a, \hat{t}) dx - d''(a) \right\} = 0. \quad (\text{FOC-a})$$

and Assumption (SOC-A). Therefore, the assumption that  $\mu \leq 0$  cannot be true. Q.E.D.

We first prove some properties of  $y(a, \hat{t})$  here.

The proof of Lemma 3:

When  $\hat{\alpha}(a, \hat{t}) \leq \delta$ ,  $\int_0^{\delta} [1 - p(a, \alpha)] d\alpha \geq \hat{t}$  and  $y(a, \hat{t})$  is defined by (1). Differentiating

$y(a, \hat{t})$  with respect to  $\hat{t}$ , we have

$$\frac{\partial y}{\partial \hat{t}} = -\frac{\partial \hat{\alpha}}{\partial \hat{t}} [r(\hat{\alpha}) - i] [1 - p(a, \hat{\alpha})].$$

Differentiating (2) with respect to  $\hat{t}$ , we have

$$1 = \frac{\partial \hat{\alpha}}{\partial \hat{t}} [1 - p(a, \hat{\alpha})].$$

Combining the above two equations yields the result.

Q.E.D.

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<sup>16</sup>This proof is adapted from Holmstrom (1979).

The proof of Lemma 4:

When  $\hat{\alpha}(a, \hat{t}) \leq \delta$ ,  $\int_0^{\delta} [1 - p(a, \alpha)] d\alpha \geq \hat{t}$  and  $y(a, \hat{t})$  is defined by (1). Differentiating

$y(a, \hat{t})$  with respect to  $a$  yields

$$\frac{\partial y}{\partial a} = -\frac{\partial \hat{\alpha}}{\partial a} [r(\hat{\alpha}) - i] [1 - p(a, \hat{\alpha})] - \int_{\hat{\alpha}}^{\delta} [r(\alpha) - i] \frac{\partial p}{\partial a} d\alpha + c \int_0^{\delta} \frac{\partial p}{\partial a} d\alpha.$$

Differentiating (2) with respect to  $a$  yields

$$0 = \frac{\partial \hat{\alpha}}{\partial a} [1 - p(a, \hat{\alpha})] - \int_0^{\hat{\alpha}} \frac{\partial p}{\partial a} d\alpha. \quad (8)$$

Substituting (8) into  $\frac{\partial y}{\partial a}$ , we have

$$\begin{aligned} \frac{\partial y}{\partial a} &= -[r(\hat{\alpha}) - i] \int_0^{\hat{\alpha}} \frac{\partial p}{\partial a} d\alpha - \int_{\hat{\alpha}}^{\delta} [r(\alpha) - i] \frac{\partial p}{\partial a} d\alpha + c \int_0^{\delta} \frac{\partial p}{\partial a} d\alpha \\ &= [c + i - r(\hat{\alpha})] \int_0^{\hat{\alpha}} \frac{\partial p}{\partial a} d\alpha + \int_{\hat{\alpha}}^{\delta} [c + i - r(\alpha)] \frac{\partial p}{\partial a} d\alpha > 0. \end{aligned}$$

Differentiating  $\frac{\partial y}{\partial \hat{t}}(a, \hat{t}) = -r[\hat{\alpha}(a, \hat{t})] + i$  with respect to  $a$  yields

$$\frac{\partial^2 y}{\partial \hat{t} \partial a} = -r'[\hat{\alpha}(a, \hat{t})] \frac{\partial \hat{\alpha}}{\partial a}.$$

(8) implies  $\frac{\partial \hat{\alpha}}{\partial a} > 0$ . Therefore,  $\frac{\partial^2 y}{\partial \hat{t} \partial a} < 0$ . Q.E.D.

The proof of Proposition 5:<sup>17</sup>

For convenience, we call an agent type  $a$  if he has chosen effort level  $a$ . Before proving the proposition, we first prove the following lemma.

**Lemma:** Given contract  $C_0$ , it is optimal for a type  $a$  agent to pick the reward scheme  $w(x, \hat{t}; a)$ .

**Proof:** By the incentive compatibility constraint (IC) in program  $P(a)$ ,

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<sup>17</sup>This proof is adapted from the proof of Proposition 1 in Ma (1994).

$$\begin{aligned} & \max_{\hat{t}} \int v(w(x, \hat{t}; a)) f(x; a, \hat{t}) dx - d(a) \geq \\ & \max_{\hat{t}} \int v(w(x, \hat{t}; a)) f(x; a', \hat{t}) dx - d(a') \text{ for all } a \text{ and } a'. \end{aligned}$$

By the individual rationality constraint (IR) in programs  $P(a)$  and  $P(a')$ , the left hand side of the above inequality is equal to

$$\max_{\hat{t}} \int v(w(x, \hat{t}; a')) f(x; a', \hat{t}) dx - d(a').$$

Therefore,

$$\max_{\hat{t}} \int v(w(x, \hat{t}; a')) f(x; a', \hat{t}) dx \geq \max_{\hat{t}} \int v(w(x, \hat{t}; a)) f(x; a', \hat{t}) dx \text{ for all } a \text{ and } a',$$

i.e. the type  $a'$  agent prefers the reward scheme  $w(x, \hat{t}; a')$  to the reward scheme  $w(x, \hat{t}; a)$  for all  $a$  and  $a'$ . Q.E.D.

Proof of the proposition: Consider any contract renegotiation offer from the agent,  $D = \{w^k(x, \hat{t})\}_{k \in K}$ . Define  $k(a) \equiv \arg \max_k \max_{\hat{t}} \int v(w(x, \hat{t}; a)) f(x; a, \hat{t}) dx$ ; <sup>18</sup> that is  $k(a)$  is the index of a type  $a$  agent's most preferred reward scheme in  $D$ .

We now describe the agent's equilibrium strategy. In subgame  $C_0$ , he takes action  $a^*$  and offers  $D = C_0$ . When choosing among reward schemes in  $C_0$ , he picks scheme  $w(x, \hat{t}; a)$  if he has taken action  $a$ . If he is to choose a reward scheme from any other contract  $D = \{w^k(x, \hat{t})\}_{k \in K}$ , he picks  $w^{k(a)}(x, \hat{t})$  if he has taken action  $a$ . He always chooses  $\hat{t}$  to maximize his expected income given the action and reward scheme that he has chosen.

Next, we describe the principal's strategy in subgame  $C_0$ . We begin by describing the principal's belief on the agent's action for any Renegotiation offer  $D$ . First, compute the set of best actions under  $D$ . That is, for any contract  $D$ , define the set

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<sup>18</sup>Since the set of reward schemes may not be finite,  $k(a)$  may not always be defined. Here we make the assumption that  $\sup_k \int v(w(x, \hat{t}; a)) f(x; a, \hat{t}) dx$  is always attained at some  $k$  so that  $k(a)$  is defined. If  $\sup_k \int v(w(x, \hat{t}; a)) f(x; a, \hat{t}) dx$  is attained at more than one point, pick any one of the points to be  $k(a)$ .

$$\Psi(D) = \{a: a \in \arg \max_a \max_{\hat{t}} \int v(w^{k(a)}(x, \hat{t}))f(x; a, \hat{t})dx - d(a)\}.$$

Let

$$a_D = \begin{cases} a^* & \text{if } a^* \in \Psi(D), \\ \sup \Psi(D) & \text{otherwise.} \end{cases}$$

The principal believes that the agent has selected action  $a_D$ . This belief is consistent with the agent's equilibrium strategy.

Given this system of beliefs and the agent's strategy, the principal chooses between contracts  $C_0$  and  $D$  to maximize her utility. She selects  $C_0$  if and only if

$$\int w(x, \hat{t}_1, a_D)f(x; a_D, \hat{t}_1)dx \leq \int w^{k(a_D)}(x, \hat{t}_2)f(x; a_D, \hat{t}_2)dx, \quad (9)$$

where,  $\hat{t}_1$  ( $\hat{t}_2$ ) is the agent's best choice of  $\hat{t}$  given action  $a_D$  and reward scheme  $w(x, \hat{t}, a_D)$  ( $w^{k(a_D)}(x, \hat{t})$ ).

We now verify that these strategies and beliefs form a perfect-Bayesian equilibrium in subgame  $C_0$ . Clearly the principal's strategy is optimal given her beliefs and the agent's strategy in subgame  $C_0$ . The agent's selection rule from any contract is also optimal. If the agent offers  $D = C_0$ , taking action  $a^*$  is optimal, since given  $C_0$  he is indifferent between all actions and attains his reservation utility.

It remains to show that the agent cannot gain by proposing  $D \neq C_0$ . Since the agent can always obtain his reservation utility by proposing  $C_0$ , we can assume that his optimal action given  $D$  is at least the reservation utility. By the assumption that  $a_D$  is one of his best action, we have

$$\max_{\hat{t}} \int v(w^{k(a_D)}(x, \hat{t}))f(x; a_D, \hat{t})dx - d(a_D) \geq \max_{\hat{t}} \int v(w^{k(a)}(x, \hat{t}))f(x; a, \hat{t})dx - d(a) \text{ for all } a.$$

Also, a type  $a$  agent prefers reward scheme  $w^{k(a)}$  to reward scheme  $w^{k(a_D)}$ . Hence

$$\max_{\hat{t}} \int v(w^{k(a)}(x, \hat{t}))f(x; a, \hat{t})dx \geq \max_{\hat{t}} \int v(w^{k(a_D)}(x, \hat{t}))f(x; a, \hat{t})dx \text{ for all } a.$$

These two inequalities imply that  $w^{k(a_D)}$  satisfies the incentive compatibility constraint (IC) and the individual rationality constraint (IR) in the program  $P(a_D)$ .

Hence (9) must hold; the principal will reject  $D$ . It follows that the agent cannot gain by proposing such a contract. Q.E.D.

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